

This recitation assignment is due at the end of your recitation 1-12-09. A key will be posted to www.stt.msu.edu/~lepage that Tuesday evening after all recitations have ended.

1. One may disagree with the following definition from page 378 since there is no notion of *probability* present in the definition.

Random phenomenon 364, 365. A phenomenon is random if we know what outcomes could happen, but not which particular values will happen.

Especially, I am troubled by the following false claim made for it on pg. 379.

► Understand that random phenomena are unpredictable in the short term but show long-run regularity.

In this course we explore many useful examples *intentionally introducing randomness* as when shuffling a deck of cards, spinning a well made roulette wheel, or randomly selecting a portfolio of investments. This differs from a deck of cards we know nothing about, spinning a toy roulette wheel, or selecting a portfolio based on the insights of a columnist. We are generally not in a position to say anything much about probabilities in the latter cases and may not be in a position to claim long run average behaviors should we be inclined to replicate many such activities.

Natural processes often appear to be random and do exhibit long run regularities. Fiddler crabs have one very large claw and another very small. In most species this "handedness," some left clawed the rest right clawed, appears in captured examples to behave like independent tosses of a slightly biased coin. We discover this from observation. Things seem to be random. The probabilities are empirical (derived from experience) although they could change if something alters the breeding population. Some such populations are around 98% right clawed.

Do you find randomness or only the possibility of differing outcomes in the examples below? If I have empirical data I may find randomness where you only see the possibility of differing outcomes. Perhaps surprisingly, you may think there is probability present where I do not.

a. On the first day of class some students sit in the first two rows. The last digits of each of their student numbers are integers in the range 0 through 9. I will look up the last digits of these students on the class list.

According to the registrar student numbers are assigned sequentially as students are admitted. So there are very close to equal numbers of last digits 0 thru 9 in the student

pool. The admissions ordering combined with the uncertainties of seating (which would seem to be unrelated to last digit) suggest that when a student is examined it is equally likely their last digit is any of 0 through 9. Furthermore, it would seem that the last digits of several students should all be statistically independent. So if, for example, I look at the last digits for several students that will tell me nothing about the last digits of other students. I've tested this model in various classes (will report on a test run for your class Wednesday) finding support for the model. So this example seems to be one with genuine randomness.

b. Same as (a). Some of these students have at least one quarter on their person. I will ask which ones do.

I do not know of any probabilities governing this situation. There may be some but I have no knowledge of that. So for this example I must say it is not known (to me) to be random.

c. A weather report says *twenty percent chance of rain tomorrow*. I will check to see if it rains tomorrow.

This one is challenging. After all, there is mention of probability. However, having spoken with some people responsible for forecasting I do not think these "probabilities" represent the actual chances of the associated events. I would need to see a lot of evidence to believe that there is genuine randomness here, as would be sufficient for long run averages to evidence themselves.

On the other hand, in a locale in which one year is very much like another we might speak about the number of rain days in a year (for example) which likely could be modelled rather well a random (more about that later in the course).

2. Engineering studies for a particular model of twin engine aircraft suggest that failure of the left engine upon landing has probability on the order of 0.0001, same as for the right engine.

a. What is the probability the left engine does not fail upon landing? See pg. 371.

Complement rule $P(\text{not } A) = 1 - P(A)$. So

$$P(\text{left engine does not fail}) = 1 - P(\text{left engine fails}) = 1 - 0.0001 = 0.9999.$$

b. Assuming that the events $A = \text{left engine fails}$, $B = \text{right engine fails}$ are *statistically independent* what is the probability that both engines fail? See pg. 373.

Multiplication rule for independent events

$$P(A \text{ and } B) = P(A) P(B) = 0.0001 \cdot 0.0001 = 0.00000001.$$

c. From (b) and the addition rule determine the probability that at least one of the two engines fails. See pg. 372 (**actually we need pg. 385**).

General addition rule pg. 385 says

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.0001 + 0.0001 - 0.00000001.$$

d. From (d) determine the probability that *it is not true that at least one engine fails*. See pg. 371.

Complement rule

$$P(\text{NOT true that at least one fails}) = 1 - P(\text{at least one fails}) \\ = 1 - (0.0001 + 0.0001 - 0.00000001).$$

e. The event that *it is not true that at least one engine fails* is the same as *the left engine does not fail and the right engine does not fail*. If engines fail *independently* this may be determined as .9999 times .9999 (why?). See pg. 373 and compare your answer with the other solution (d).

Events "left does not fail" and "right does not fail" are likewise independent. So

$$P(\text{neither fails}) = P(\text{left does not fail}) P(\text{right does not fail}) \\ = 0.9999 \cdot 0.9999.$$

Use your calculator (or a little algebra) to see that the answers (d) and (e) are exactly the same. We've calculated the probability that neither engine fails in two different ways.

f. Suppose engine failures are mainly caused by flocks of birds some of whom enter the engine air intake. Would this be consistent with engines failing independently?

Probably not independently. Failure of the left engine points to birds as the cause which in turn makes failure of the right engine more probable that it would have been without knowing birds were present upon landing. When would engine failures be independent? In situations where there are no predominating causes of failure that simultaneously affect both engines (absence of such things as flocks of birds, dust storms or other extreme weather conditions, electrical failures, fueling errors or fuel contamination, etc.). Independence is when the causes are isolated and rarely encountered component failures like a blade breaking off, the rare big bird, etc.

g. Are the events *left engine fails* and *right engine fails* disjoint? Why?

They would be disjoint if they could not simultaneously occur. Since both engines can fail on the same flight these events are not disjoint. Were they disjoint then $P(\text{both fail}) = 0$ which would imply

$$P(\text{at least one fails}) = 0.0001 + 0.0001 - 0.$$

That is NOT the case here since $0 < P(\text{both fail})$ (e.g. would be $0.0001 \cdot 0.0001$ if failures of the left and right engines were independent events).

3. Here are counts of people cross classified by gene type and sex.

	AA	Aa	aa	
male	20	30	10	60
female	30	45	15	90
	50	75	25	150

a. How many people are there in all?

150

b. Determine the marginal counts for males and females (there are 60 males) and place them as row totals to the right margin of the table.

c. Determine the marginal counts for gene types (there are 50 who are AA) and place them as column totals to the bottom of the table.

d. Place the grand total (a) to the lower right corner of the table.

Suppose we are to sample **one of these 150** people, each one of them having an equal chance to be selected. We are *introducing* probability. All individuals are equally likely so a *classical probability model will apply*.

e. If John A. F. Hampel is in the group what is the probability we select him?

1/150 assuming the name to be unique in the 150.

f. What is the probability that we select a male?

60/150

g. What is the probability that we select a person with gene type AA?

50/150

h. What is the probability that we select a male having gene type AA?

20/150

i. Are the events
 select a male
 select an AA
 disjoint events?

No, both can occur in the sample person.

j. Are the events
 select a male select an AA
independent events?

Check the definition. $P(\text{male AA}) = 20/150$. Is this equal to the product of marginal probabilities $P(\text{male}) P(\text{AA})$? The latter is $60/150 \cdot 50/150 = 3000 / (150 \cdot 150) = 20/150$. So yes, these are independent events.

k. We say sex is independent of gene type if all six of the sex by gene type pairs are independent. Is it so for this table?

Check that indeed all event pairs such as (male, aa) etc. are likewise independent. So yes, sex is independent of gene type.

l. Look carefully at the table. Is there anything special about its numbers that betray independence?

The rows are in constant proportion 20:30 = 30:45 = 10:15 so yes, sex is independent of gene type for this data just by observation of this fact.

4. Here are all 36 ways that a red and green die might turn up. By looking at favorable cases determine the probability that the total of the numbers thrown by the dice is 5. Assume all 36 outcomes are equally likely and check off favorable cases.

	G	1	2	3	4	5	6
R							
1					*		
2				*			
3			*				
4		*					
5							
6							

$$P(R+G = 5) = \frac{\# \text{ favorable}}{\# \text{ total}} = 4/36$$

5. A box contains three bills \$1 \$1 \$5. Jack will draw first with equal probability on the three. Jill will then draw with equal probability from the two then remaining.

a. What is the probability that Jack gets 5? Answer on an intuitive basis.

1/3 using the model of a single draw from 1, 1, 5.

b. What is the probability that Jill gets \$5? Answer on an intuitive basis.

Not obvious to many people but also $1/3$. We want to get Jack into the act since we know what he does influences Jill's chances. The way to see this (taken up in chapter 15) is that Jill gets 5 precisely when Jack gets 1 and Jill gets 5.

$$P(\text{Jack 1}) = 2/3$$

$$P(\text{Jill 5 if Jack 1}) = 1/2 \text{ (Jill wins 5 in half of the cases Jack 1).}$$

So chance Jill 5 = chance Jack 1 and Jill 5 = $2/3 \cdot 1/2 = 1/3$.

Generally, in a classical model

$$P(\text{A and B}) = \frac{\# A \cap_{\text{and}} B}{\# \text{total}} = \frac{\# A}{\# \text{total}} \frac{\# A \cap_{\text{and}} B}{\# A}$$

def P(A) our def of P(B | if A)

$$2/3 \quad 1/2 \quad = \quad 1/3$$

c. We are actually drawing pieces of paper. Consider a classical probability model as follows. Label the papers a, b, c (c is the \$5). We regard all of the following equally probable:

Jack	Jill	
a	b	
a	c	*
b	a	
b	c	*
c	a	
c	b	

So $P(\text{Jill } \$5) = 2/6 = 1/3$.

By checking off favorable cases determine the probability Jack gets \$5.

By checking off favorable cases determine the probability Jill gets \$5.

Are your answers just obtained in agreement with your intuitive ones?

Do you believe the answers of (c) are the correct ones?