

Recitation assignment Chapter 15. Questions 1-4 due 1 - 19 - 10 and 5, 6 due 1-26-10 along with an assignment from Chapter 16.

Don't underestimate this material.

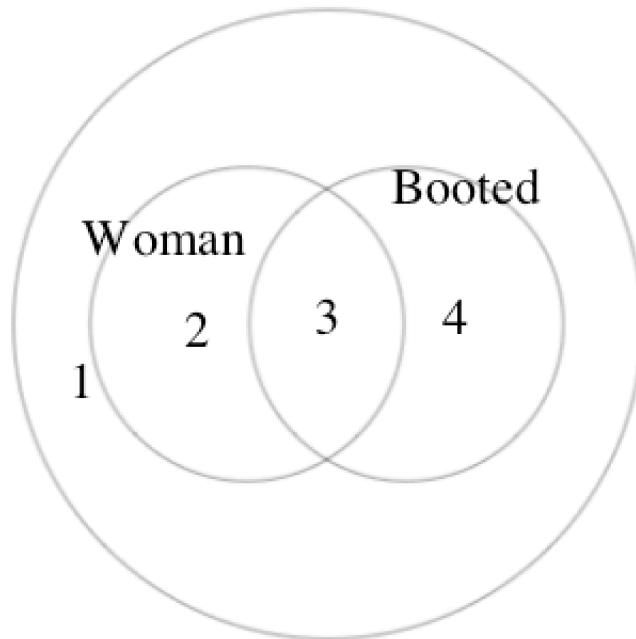
The rules of probability actually embody several simple principles of counting.

a. **Addition Rule:** Embodies the idea that if we count the number of **booted students in the class** then add to that the number of **women in the class** and finally subtract the number of **booted women in the class** we get the number of students who will answer yes to the question "**are you a woman or are you wearing boots?**" Picture this in a Venn diagram (you can use the index to find it). In probability terms

$$P(\text{woman or booted}) = P(\text{woman}) + P(\text{booted}) - P(\text{booted woman}).$$

$$P(\text{Woman or Booted}) = P(\text{Woman}) + P(\text{booted}) - P(\text{Both})$$

$$2 + 3 + 4 \qquad 2 + 3 \qquad 3 + 4 \qquad 3$$



b. **Multiplication Rule:** Embodies the idea that if 20% of the class are women and 60% of women in the class are booted then 12% of the class are booted women. It is simpler with

fractions $0.2 \times 0.6 = 0.12$. Work it out for (say) 100 students of which 20 are women and of these 20 women 60% or 12 are booted. So booted women comprise 12% of the class. Since it is a universal rule the example of 100 students must give the correct answer. In probability terms:

$$P(\text{booted woman}) = P(\text{woman}) P(\text{booted if a woman})$$

$$P(\text{booted} \cap_{\text{and}} \text{woman}) = P(\text{woman}) P(\text{booted} \mid_{\text{if}} \text{a woman}) \\ = 0.2 \quad 0.6 \quad = 0.12$$

c. Independence: If the rate of wearing boots is the same among women as it is among men then we learn nothing about whether the person is wearing boots from noting the sex. Likewise we learn nothing about the sex upon noting that boots are worn. In probability terms **independence** means:

$$P(\text{booted} \mid_{\text{if}} \text{a woman}) = P(\text{booted} \mid_{\text{if}} \text{a man}) = P(\text{booted})$$

which has the consequence that

$$P(\text{booted and woman}) = P(\text{woman}) P(\text{booted} \mid_{\text{if}} \text{a woman}) \text{ (see pg. 391)} \\ = P(\text{woman}) P(\text{booted})$$

1. **Independent events A, B are said to have $P(A) = 0.8$, $P(B) = 0.7$.**

a. $P(A \cap_{\text{and}} B) = P(A) P(B \mid A) = (\text{if independent}) P(A) P(B) = 0.8 \cdot 0.7 = 0.56$

Independence means that nothing learned about the occurrence or non- occurrence of one event causes us to revise our probability for the other.

b. $P(B \mid_{\text{if}} A) = (\text{if independent}) P(B) = 0.7$

c. $P(A \mid_{\text{if}} B) = (\text{if independent}) P(A) = 0.8$

d. $P(B \mid_{\text{if}} \text{not } A) = (\text{if } A, B \text{ are independent so are } A, B^C) P(B) = 0.7$

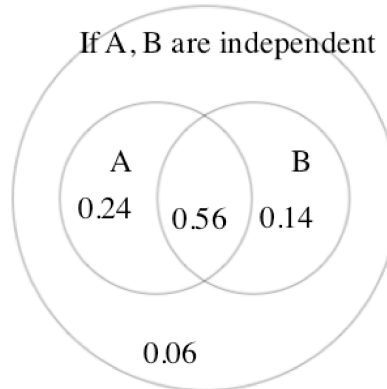
e. $P(B \mid_{\text{if}} A^C) = P(B) = (\text{if independent}) 0.7$

f. $P(B \mid \bar{A}) = P(B) =$ (if independent) 0.7 (parts d, e, f ask for the same thing)

$$P(A \text{ and } B) = 0.8 \cdot 0.7 = 0.56$$

$$P(\text{not}A \text{ and } \text{not}B) = 0.2 \cdot 0.3 = 0.06$$

g. Fill out a **complete** Venn Diagram.



$$P(A \text{ and } \text{not}B) = 0.8 \cdot 0.3 = 0.24$$

$$P(\text{not}A \text{ and } B) = 0.2 \cdot 0.7 = 0.14.$$

h. Determine $P(A \cup_{\text{or}} B)$.
 $0.24 + 0.56 + 0.14$

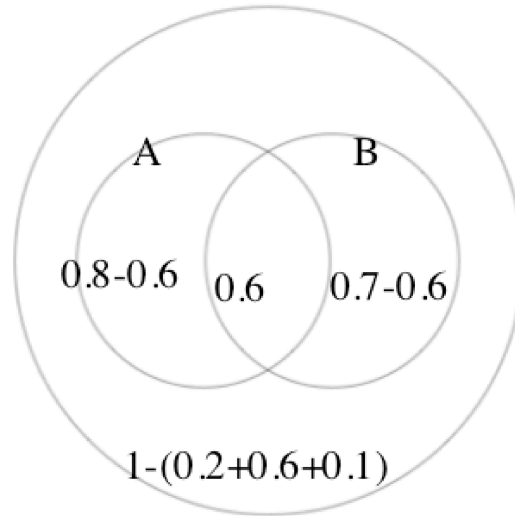
2. Events A, B are said to have $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cap_{\text{and}} B) = 0.6$.

a. Are A, B **independent events**? Why? NO, $0.6 \neq 0.8 \cdot 0.7$ (which is 0.56).

b. Determine $P(B \mid A)$ using the definition on pg. 390.
 $P(A \text{ and } B) / P(A) = 0.6 / 0.8 = 0.75$

c. Events **A**, **B** are said to have $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cap_{\text{and}} B) = 0.6$. Verify the multiplication rule of pg. 391 $P(A \cap_{\text{and}} B) = P(A) P(B \mid_{\text{if}} A)$. It is just the definition of **conditional probability rearranged**. $0.6 = 0.8 \cdot 0.75$

d. Fill out a **complete** Venn diagram.



e. Determine $P(A \cup_{\text{or}} B)$. See pg. 385. $0.2 + 0.6 + 0.1$

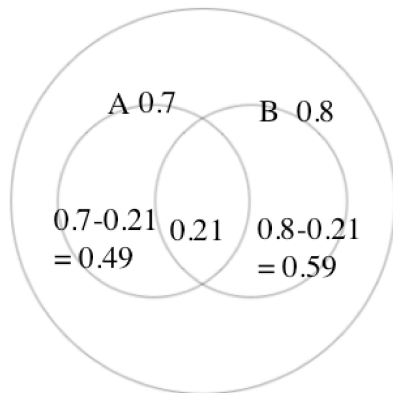
f. Observe that the marginal probabilities $P(A)$, $P(B)$ are the same for problems #1 and #2 but the probability of their overlap $A \cap_{\text{and}} B$ differs for the two examples as do conditional probabilities such as $P(B \mid_{\text{if}} A)$. YES

g. Determine the reverse conditional probability $P(A \mid_{\text{if}} B)$ from the Venn diagram (d) as the ratio of the probability of the overlap to the probability of the conditioning event **B** (in this case). See that this is just the definition of pg. 390.

$$P(A \mid B) = P(A \text{ and } B) / P(B) = 0.6 / 0.7$$

3. Events **A**, **B** are said to have $P(A) = 0.7$, $P(B) = 0.8$ and $P(B \mid A) = 0.3$. By trying to fill out the Venn diagram for this information see this is not possible. Some claims just cannot be true.

$$P(A \text{ and } B) = P(A) P(B \mid A) = 0.7 \cdot 0.3 = 0.21$$



But $0.49 + 0.21 + 0.59 > 1$ (impossible).

4. Here are counts of people cross classified by gene type and sex.

	AA	Aa	aa
male	30	20	10
female	30	45	15

Suppose we are to sample from these people, each one of them having an equal chance to be selected. We are *introducing* probability. Since all individuals are equally likely this is an example of a *classical probability model*. Fill out the cells and margins with the applicable probabilities. For example, $P(\text{male and AA}) = 30 / 150$, $P(\text{male}) = 60 / 150$, $P(\text{AA}) = 60 / 150$.

	AA	Aa	aa	
male	30/150	20/150	10/150	60/150
female	30/150	45/150	15/150	90/150
				1.0

- a. Determine $P(\text{AA} \mid \text{male})$. $P(\text{AA and male}) / P(\text{male})$
 $= (30/150) / (60/150) = 30/60 = 0.5$
- b. Verify that $P(\text{male and AA}) = P(\text{male}) P(\text{AA} \mid \text{male})$.
 $30/150 = 60/150 \cdot 30/60$ (the 60 cancels)

- c. Determine $P(\text{male} \mid \text{AA}) = P(\text{AA and male})/P(\text{AA}) = 30/60$
 (in this case the same as $P(\text{AA} \mid \text{male})$ but that is an accidental fact)
- d. Verify that $P(\text{male and AA}) = P(\text{AA}) P(\text{male} \mid \text{AA})$.

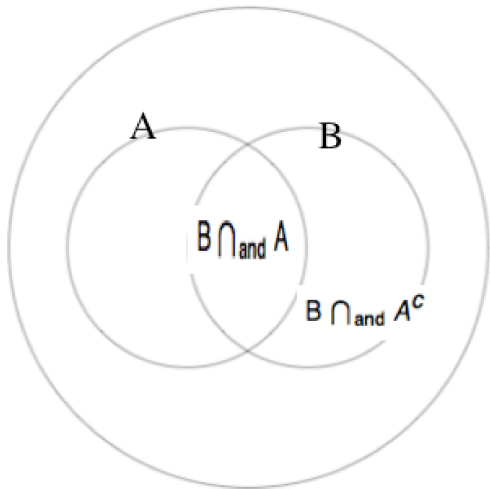
$$\frac{30}{150} = \frac{60}{150} \cdot \frac{30}{60}$$
- e. Are events male, AA independent?
 No, for example 20:10 is not the same as 45:15 (table not proportional).

5. Jack and Jill will draw from the box with bills \$1 \$1 \$5. Jack draws first with equal probability. Jill draws second with equal probability from the two bills then remaining. Last week we found that $P(\text{Jill gets } \$5) = 1/3$, same as for Jack. This seems a puzzle for we intuitively believe Jack influences Jill's chances. Indeed so.

- a. Intuitively, what is $P(\text{Jill } \$5 \mid \text{Jack } \$5)$? Zero!
- b. Intuitively, what is $P(\text{Jill } \$5 \mid \text{Jack } \$1)$? $1/2$
 (since then Jill draws from \$1 \$5)
- c. The **law of total probability** says

$$P(B) = P(B \cap A) + P(B \cap A^c)$$

Illustrate this by shading the affected regions of the Venn diagram.



d. Using (c) we can calculate $P(\text{Jill } \$)$ as follows:

$$\begin{aligned} P(\text{Jill } \$5) &= P(\text{Jill } \$5 \text{ and Jack } \$5) + P(\text{Jill } \$5 \text{ and Jack } \$1) \\ &= 0 + P(\text{Jill } \$5 \text{ and Jack } \$1) \\ &= P(\text{Jack } \$1) P(\text{Jill } \$5 \mid \text{if Jack } \$1) \\ &= \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3} \text{ (same chance as for Jack)} \end{aligned}$$

Justify these steps and using your answers to (a) (b) to determine $P(\text{Jill } \$5)$. The take-away message is the rules can be used without enumerating cases as we had to do in chapter 14.

So yes, Jack does influence what happens to Jill, but averaged over the chances for Jack Jill looks OK. Anyway, we don't fight over who sits next to the dealer at cards do we? So why think Jack has an advantage dealing to him first from the cards 1 1 5?

This kind of probability thinking is not part of our evolutionary development?

6. We are told that

$P(\text{OIL}) = 0.2$ $P(\text{test is } + \mid \text{if OIL}) = 0.9$ $P(\text{test is } + \mid \text{if no OIL}) = 0.3$
that last one is called a "false positive." Determine

a. $P(\text{OIL}^c) = 1 - P(\text{OIL}) = 1 - 0.2 = 0.8$

b. Determine $P(\text{test is } - \mid \text{if OIL}) = 0.1$ (just $1 - 0.9$)

c. Determine $P(\text{OIL}+) = P(\text{OIL}) P(+ \mid \text{if OIL})$
 $= 0.2 \cdot 0.9 = 0.18$

d. Determine $P(+)$ by summing $P(\text{OIL}+) + P(\text{OIL}^c+)$, obtaining these from

$$P(\text{OIL}+) = P(\text{OIL}) P(+ \mid \text{if OIL}) = 0.18$$

$$P(\text{OIL}^c+) = P(\text{OIL}^c) P(+ \mid \text{if OIL}^c) = 0.8 \cdot 0.3 = 0.24$$

e. Fill out a complete tree diagram for the given information. (pg. 396)

OIL 0.2	+ 0.9	OIL + 0.2 0.9 = 0.18
	- 0.1	OIL - 0.2 0.1 = 0.02
OIL ^c 0.8	+ 0.3	OIL ^c + 0.8 0.3 = 0.24
	- 0.7	OIL ^c - 0.8 0.7 = 0.56
total = 1		

(you get Venn from the column above ↑↑)

f. Determine $P(\text{OIL} \mid \text{if } +)$. That is, the **revised probability** for OIL if the test for it is positive (which would seem to increase the chance for OIL). Use the rules to determine it as:

$$P(\text{OIL} \mid \text{if } +) = \frac{P(\text{OIL and } +)}{P(+)} = \frac{\text{answer (c)}}{\text{answer (d)}} = \frac{0.18}{0.18 + 0.24} = 3/7 = 0.42857.$$

This kind of calculation (pioneered by Rev. Bayes) enables us to update probabilities to take account of new information. In this example we originally had probability 0.3 for OIL. After a positive test finding this probability is increased to the value you get in (f).