

Recitation assignment 1-26-10. Questions 5-6 of assignment 1-19-10 and the following.

1. A box has colored balls numbering 5R 2G 8Y. A ball is selected from the 15 with equal probability for all balls. Then a second ball is selected from the remaining balls with equal probability. This is termed "without replacement" sampling. Determine

a.  $P(\text{first ball is red}) = P(R_1) =$

b.  $P(\text{first ball is red and second ball is red})$   
 $= P(R_1 R_2) = P(R_1) P(R_2 | \text{if } R_1) =$

c.  $P(\text{first ball is not red and second ball is red})$  note correction  
 $= P(R_1^c R_2) = P(R_1^c) P(R_2 | \text{if } R_1^c) =$

d.  $P(\text{second ball is red}) = P(R_1 R_2) + P(R_1^c R_2) =$   
 (law of total probability, you get R2 by exactly one of R1 or R1<sup>c</sup>)

e. Compare  $P(R_2)$  with  $P(R_1)$ . Is one larger than the other?

2. A person is sampled at random from a population. Let D be the event that the person has a particular disease. Let + be the event that the person tests positive for the disease. Suppose an applicable Tree Diagram is:

	+	0.936	D+ (i.e. D and +)
D 0.0001	-		D-
	+		D <sup>c</sup> +
D <sup>c</sup>	-	0.905	D <sup>c</sup> -

a.  $P(D^C) =$

b.  $P(+ \mid \text{if } D) =$

c.  $P(D+) =$

d.  $P(D^C+) =$

e.  $P(+) =$

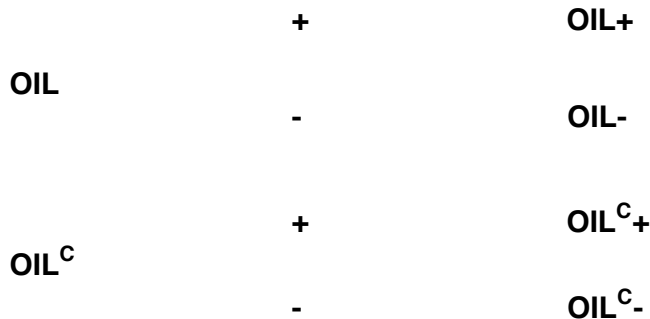
f.  $P(D \mid \text{if } +) =$

g. Which of  $D+$ ,  $D-$ ,  $D^C+$ ,  $D^C-$  are the "false positive" and "false negative?" Give their respective probabilities.

h. Are you surprised by the value of  $P(D \mid \text{if } +)$ ? Even a good test for disease (one having low probabilities of false positive or false negative) may be overwhelmed and produce a lot of false positives if we administer it to lots of healthy people. In a panic the test becomes worthless.

3. Suppose  $P(OIL) = 0.2$ ,  $P(+ | \text{if } OIL) = 0.9$ ,  $P(+ | \text{if } OIL^C) = 0.3$ .

a. Make the complete tree diagram (with all probabilities).



Suppose also that it costs 200 to drill, 80 to test, and the gross return from finding oil is 900.

If we adopt the policy "just drill, do not test" the possible NET returns and their probabilities are:

net return x	900-200	-200
probability p(x)	0.2	0.8

b. Calculate the expected net return from policy "just drill." That is, calculate the sum of (value x probability). This is written

$$E X = \sum x p(x).$$

See lecture 1-20-10 and also pg. 411.

If we adopt the policy "pay to test but only pay to drill if the test yields a positive result" we have four contingencies OIL+, OIL-, OIL<sup>C</sup>+, OIL<sup>C</sup>-. The net return from this policy for contingency D+ is

$$- 80 \quad - 200 \quad + 900 \quad = 620$$

test    drill because test is +    find OIL    net return for contingency D+

The probability of the contingency OIL+ is  $P(OIL) P(+ | \text{if } OIL) = 0.2 \cdot 0.9 = 0.18$ .

Find net returns for all of the four contingencies above and their respective probabilities:

contingency	OIL+	OIL-	OIL <sup>C</sup> +	OIL <sup>C</sup> -
net return x	620			
probability	0.18			

- c. Calculate the expected net return  $x$  for the policy "test, but only drill if the test is positive."
- d. Which policy has the larger expected net return?

4. **A lottery offers returns  $x$  having the probabilities  $p(x)$  as shown below.**

$x$	20	-5	0	
$p(x)$	0.2	0.4		<b>0.4 note correction</b>

- a. Calculate  $E X = \sum x p(x) =$
- b. Calculate Variance  $X = E (X - E X)^2 = E X^2 - (E X)^2$  (see Lecture 1-20-10 and pp. 412-413).
- c. Calculate Standard Deviation of  $X = \sqrt{\text{Variance } X}$  (see Lecture 1-20-10 and pp. 413-414).

5. **A lottery has expected return \$-0.10 (loses ten cents on average for one play). The standard deviation of  $X$  is \$0.80. If you play this lottery 100 times (independent plays) and  $T$  denotes the total of the 100 random returns,**

- a. Give  $E T$  (expected total of 100 independent play, see Lecture 1-20-10 and pg. 418).
- b. Give Variance  $T$  and Standard Deviation of  $T$  (see Lecture 1-20-10 and pg. 418).
- c. Sketch the bell curve showing the approximate appearance of a lot of possible values  $t$  (of  $T$ ) together with their probabilities (see Lecture 1-20-10).