

Recitation assignment due in recitation 2 - 2 - 10. See chapters 16 and 17.

Reminder: Exam 1 is 2-3-10. Lecture 1-27-10 will be very important in getting you started with the material below.

Errata:

On page 426 it is said that "410, 420. A random variable that can take any numeric value within a range of values is called a continuous random variable.." This is incorrect. To see the difficulty imagine that I flip a coin and if heads occurs I say $X = 1$, otherwise I spin a pointer and announce X as the angle, in infinite precision degrees 0 to 360, at which the pointer comes to rest. X may take any value in the interval $[0, 360]$ but is not continuous since there is discrete probability 0.5 that X takes the value 1. See lecture notes 1-27-10.

On page 426 it is said that "410. The probability model is a function that associates a probability P with each value of a discrete random variable X , denoted $P(X = x)$, or with any interval of values of a continuous random variable." They should have said "The probability model for a random variable X .. ." To see the difficulty consider the probability model usually specified for the toss of a coin which consists of the set $\{H, T\}$ of possible outcomes having respective probabilities $\{0.5, 0.5\}$. If you code the model numerically such as outcomes $\{0 \text{ for } T, 1 \text{ for } H\}$ with respective probabilities $\{0.5, 0.5\}$ you have another probability model whose possible outcomes $\{0, 1\}$ are numeric. This model is what the authors are talking about in 410. This model is also called the probability distribution of random variable X . We must not make the oversight of ruling out $\{H, T\}, \{0.5, 0.5\}$ as a probability model and we must recognize that we are talking about a probability distribution on the real line commonly referred to as the probability distribution of a random variable X .

1-12. Random variable X has the following distribution (some useful calculations are shown):

x	$p(x)$	$x p(x)$	$(x - (-0.7))^2 p(x)$	$x^2 p(x)$
0	0.2	0	0.49 0.2	0 0.2
1	0.3	0.3	2.89 0.3	1 0.3
-2	0.5	-1.0	1.69 0.5	4 0.5
totals	1.0	- 0.7	1.81	2.3

Confirm all calculations of the table before proceeding.

1. Determine $E X =$
(see pg. 411)
2. Directly read Variance X from the table =
(see pg. 413)
3. Calculate Variance $X = E X^2 - (E X)^2 =$
(see that your answer agrees with #2)
4. Determine $E (3 X + 7) =$
(see pp. 414-415)
5. Determine $E (3 X - X - 6) =$

6. Determine Variance of $(3 X + 7) =$
(use rules, see pg. 415)
7. Determine Variance of $(3 X - X + 7) =$
(merge $3 X$ with $-X$ first)
8. If Y is a random variable with $E Y = 6$ determine $E(X - 2Y + 4) =$
(use rules, see pg. 415)

9. If Y is a random variable **independent of** X with Variance $Y = 2$ determine Variance of $(5 X - Y + 11) =$
(see pg. 416)

10. Determine the standard deviation of random variable X (written SD or "sigma" or σ , or σ_X when we wish to specify which random variable it is the standard deviation of.
(see pg. 426)

11. Determine the SD of random variable $(3X - 2X + 4) =$
(use the rules, see pg. 426, be careful since $3X$ and X are not independent!)

12. Refer to #9. Determine the standard deviation of $(5 x - Y + 11)$.
(use rules, see pg. 426)

13-17. Lottery 1 returns random variable X having expectation 17 and variance 4. Lottery 2 returns random variable Y having expectation 30 and variance 9. We are invited to play each of these but it will cost us 2 to play for X and 3 to play for Y . As a bonus, we will earn $1.4X$ instead of X .

13. In terms of X , Y , 2, 3, 1.4 express a random variable describing our actual NET return R if we accept the offer.

14. Using the rules determine $E R =$

15. Using the rules determine Variance of R if X , Y are **independent**.

16. From #15 give the SD of $R =$

17. If each of X , Y follows a normal distribution the so will R (provided X , Y are independent, see pg. 422). Assuming that R follows a normal distribution sketch the distribution with the mean and SD in place and determine a 68% Interval around the mean.

Refer to chapter 17. Before exam 1 we will cover only the definition of Bernoulli Trials, the binomial distribution and its normal approximation, and the Poisson distribution and its normal approximation.

18-22. A fair coin will be tossed 100 times (any H counts as a "success").

18. What are the number n and probability p of Bernoulli trials?
(see pg. 433)

19. Define random variable X as the number of heads seen in executing #18. Determine $E X =$
, Variance $X =$, SD $X =$

20. Verify the condition (pg. 439) whereby we may approximate the distribution of X by a normal and sketch the normal approximation of the distribution of X labels included.

21. Use #20 to fill out the following:

$$P(X \text{ falls in the range } [\quad , \quad]) \sim 0.68$$

$$P(X \text{ falls in the range } [\quad , \quad]) \sim 0.95$$

$$P(\text{ we get between 40 and 55 heads in 100 tosses }) \sim$$

22. Determine the approximate 68% interval for the number of heads in 10,000 tosses of a fair coin.

23-27. A fair six-sided die will be tossed 100 times (any "ace", i.e. face 1 turning up, will be counted as a "success").

23. What are the number n and probability p of Bernoulli trials?
(see pg. 433)

24. Define random variable X as the number of aces seen in executing #23. Determine $E X =$
, Variance $X =$, SD $X =$

25. Verify the condition (pg. 439) whereby we may approximate the distribution of X by a normal and sketch the normal approximation of the distribution of X labels included.

26. Use #25 to fill out the following:

$$P(X \text{ falls in the range } [\quad , \quad]) \sim 0.68$$

$$P(X \text{ falls in the range } [\quad , \quad]) \sim 0.95$$

27. Determine the approximate 68% interval for the number of aces in 10,000 tosses of a fair die.

28-31. A hospital averages around 4.7 emergency admissions for eye injury per night. Past experience indicates that these counts X of rare events are acceptably modelled by the Poisson distribution.

28. Determine $E X =$

29. Determine $SD X =$

30. Since $E X \geq 3$, sketch the normal distribution approximating the distribution of $X = \#$ admitted with eye injury in a given night. Label mean and SD.

31. Determine a 95% interval for X . Would you be surprised to see so many as 9 admissions for eye injury in a given night? Why?