Chapter 21 supplement for Monday 2-22-10.

- **We cover only the bare essentials of Chapter 21 as previously announced. This supplement outlines a procedure applicable to the task of prescribing (in advance of sampling) the sample size n required if we specify the type 1 error and also (at a specified single point of the alternative hypothesis) the type 2 error.**
- **Notation:**

Null hypothesis is $H_0 : p \leqslant p_0$ where p_0 is some specified value strictly between zero and one.

Alternative hypothesis is $H_1 : p > p_0$.

Form of test: Reject H_0 if the number of successes in n trials is at least c (i.e. reject if $X \geqslant c$). The sample size n and integer c are to be determined.

Type 1 error probability is $P(X \geq c)$ calculated for $p = p_0$ approximated by $P(Z > (c - 0.5 - n\, p_0) / \sqrt{n\, p_0\, q_0})$ which we set equal to some desired value $P(Z > z_0)$. This gives one equation

$$(c - 0.5 - n\, p_0) / \sqrt{n\, p_0\, q_0} = z_0 .$$

Type 2 error probability is $P(X < c)$ calculated for some fixed point $p = p_1$ in the alternative hypothesis (i.e. a point larger than p_0) approximated by $P(Z < (c - 0.5 - n\, p_1) / \sqrt{n\, p_1\, q_1})$ which we set equal to some desired value $P(Z < z_1)$. This gives a second equation

$$(c - 0.5 - n\, p_1) / \sqrt{n\, p_1\, q_1} = z_1 .$$

The two equations may be solved for the required sample size n and the value c needed to conduct such a test:

$$n = \left(\frac{\sqrt{p_0\, q_0}\; \left| z_0 \right| + \sqrt{p_1\, q_1}\; \left| z_1 \right|}{p_0 - p_1} \right)^2$$

$$c = z_0 \sqrt{n\, p_0\, q_0} + 0.5 + n\, p_0$$

Example :

We need to construct a test that will randomly sample a shipment of parts finding the (random) number X of defective parts in the sample. The null hypothesis for the test (agreed upon by the parties) is

H_0**: p ≤ 0.1 (i.e. the null hypothesis is rate of defective parts is** $p = p_0 = 0.1$**).**

At issue are the sample size n and cut point c (an integer) so that
the type 1 error rate is 0.05
the type 2 error rate is 0.04
That means
the probability of rejecting a shipment having 0.1 defectives rate is 0.05
the probability of failing to reject a shipment having 0.15 defectives rate is 0.04

The type 1 and type 2 goals specified just above are for example only and would be negotiated among the parties.

To solve for n and c of the plan we need to find the values z_0 **and** z_1
$P(Z > z_0) = 0.05$ **for** $z_0 = 1.645$ **(insert 0.95 to body of z-table and read off** z_0**)**
$P(Z < z_1) = 0.04$ **for** $z_1 = -1.75$ **(insert 0.04 to body of z-table and read off** z_1**)**
For convenience you could have inserted 0.05 to the z-table getting the negative -1.645 but it will give the same n because only the absolute value of z_0 **is used. From the formulas above**

$$n = \left(\frac{\sqrt{p_0\, q_0}\, \left| z_0 \right| + \sqrt{p_1\, q_1}\, \left| z_1 \right|}{p_0 - p_1} \right)^2 = \left(\frac{\sqrt{0.1 \times 0.9}\, \left| 1.645 \right| + \sqrt{0.15 \times 0.85}\, \left| -1.75 \right|}{0.1 - 0.15} \right)^2 = 500.3$$

It is customary to round up to n = 501 since n must be an integer. As seen below, we will keep the decimal 500.3 for the calculations leading to c.

The value for c (be sure the sign of z_0 **correct) is then calculated**

$$c = z_0 \sqrt{n\, p_0\, q_0} + 0.5 + n\, p_0 = 1.645 \sqrt{500.3\ 0.1\ 0.9} + 0.5 + 500.3\ 0.1 = 61.568$$

For the course we will agree to round up to c = 62.

- **Summing up :**

We specified a test of the hypothesis of defectives rate 0.1 versus the alternative hypothesis of larger defective rate. We also specified that

the probability of our test rejecting a shipment with 10% defectives is 0.05

the probability that our test fails to reject a shipment with 15% defectives is 0.04.

The normal approximations utilized above gave sample size n = 501, the shipment to be rejected if a sample of this size finds at least 62 defectives.

Here is a sample of 501. The shipment (a large population) actually contains 13% defectives that are signified by DEF where good parts are signified by blank spaces. Carry out the test just described. What action is taken (reject or fail to reject the shipment)? Is it the correct action for this data?

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Exercise. Include this exercise in your recitation assignment for 2-23-10.

A large screening for immunization is undertaken using random sampling of the population. Design a test of the hypothesis that 17% of the population require immunization versus the alternative hypothesis that more than 17% require immunization. Specify the type 1 error rate as 3% and the type 2 error rate as 6% if 21% of the population require immunization.

Determine n, c. Be sure to identify your z-values and display your calculations as I have above. When two z-table entries are equally close use the one having the larger absolute value for z (this results in the larger sample size n).

Here is a sample of the population having the required n. Conduct the test and state the decision taken. The actual population rate of those requiring immunization is 17.8%. Has the test made a correct decision?

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