

H

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due a start of class 8-2-10.

1. Let X = the number of tosses to obtain the first head.

a. Guess $\mu = E X$ (it is intuitive)

2

$$\leadsto \mu = \frac{1}{2}(1) + \frac{1}{2}(1+\mu)$$

#1 T1

$$\Rightarrow \mu = 2$$

b. Can you guess σ ?

1

c. Let x_1 denote the number of tosses you have to make to get the first head. Repeat the experiment to get x_2 (the number of tosses you have to make to get the first head the second time you try the experiment). Do this 30 times getting x_1, \dots, x_{30} . Record the results (number of tosses required for each of 30 replications of "tossing until the first head.")

4, 3, 1, 6, 1, 1, 2, 1, 1, 1, 3, 1, 1, 2, 3, 4, 1, 4, 1, 2, 1, 1, 1, 3, 4, 3, 4, 2, 2, 5. / 30

1=13 4=5
 2=5 5=1
 3=5 6=1

d. From your sample of $n = 30$ give

69/30 = 2.3

\bar{x} (sample mean), an estimate of μ

s. your estimate of σ

$$\sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(4-2.3)^2}{29} + \dots + \frac{(5-2.3)^2}{29}}$$

1 = .058	4 = .0997
2 = .003	5 = .251
3 = .017	6 = .472

$$= 1.9178 = 1.385$$

$\frac{s}{\sqrt{n}}$, your estimate of the standard deviation of \bar{x}

$$\frac{1.385}{\sqrt{30}} = .253$$

MOE (margin of error for \bar{x}) = $1.96 \frac{s}{\sqrt{n}}$

$$1.96 \left(\frac{1.385}{\sqrt{30}} \right) = .496$$

E. McBride

08%

95% z-based CI for μ

$$\begin{array}{r} + \quad 2.3 + .253 = 2.553 \\ - \quad 2.3 - .253 = 2.047 \end{array}$$

If μ is not in your interval then a "bad" event has occurred.

What is the probability of this "bad" event?

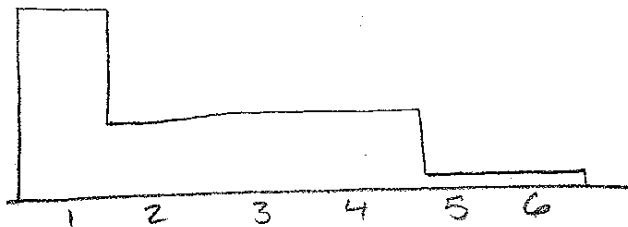
~~0.05 (the remaining portion of the curve)~~
 In class, 7/13 did not cover. = .54 w/ 68%, 32% would not cover

Around what fraction of the class should have an 80% t-CI containing μ ?

~ 80% OF CLASS

Prepare a histogram of your 30 numbers, does it look at all as though X is normal distributed?

no.



2. Let X = the number of heads in 10 tosses of a coin. Although X is not normally distributed (it is binomial) the distribution is not far from normal with mean np , and standard deviation $\sqrt{np(1-p)}$. For $n = 3$ times toss a coin 10 times recording the number of heads x_1, x_2, x_3 in each of the three experiments.

$$\begin{array}{l} x_1 = \text{||||} \quad x_1 = 4 \\ x_2 = \text{||||} \quad x_2 = 5 \\ x_3 = \text{||||} \quad x_3 = 6 \end{array}$$

From your sample of $n = 3$ give

\bar{x} (sample mean), an estimate of μ

$$15/3 = 5$$

s, your estimate of σ

$$\begin{aligned} \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} &= \sqrt{\frac{(4-5)^2}{2} + \frac{(5-5)^2}{2} + \frac{(6-5)^2}{2}} \\ &= \sqrt{\frac{1}{2} + 0 + \frac{1}{2}} = \sqrt{1} = 1 \end{aligned}$$

$\frac{s}{\sqrt{n}}$, your estimate of the standard deviation of \bar{x}

$$\frac{1}{\sqrt{2}} = .707$$

df $3-1 = 2$

t-MOE (margin of error for \bar{x}) = $t_{0.025} \frac{s}{\sqrt{n}}$

$$4.303(.707) = 3.04$$

80% t-based CI for μ

$$5 - (1.886)(1) = 3.114 \quad 5 + (1.886)(1) = 6.886$$

If μ is not in your interval then a "bad" event has occurred.
What is the probability of this "bad" event? $\mu = 5$

w/ 80% CI, 20% would not cover

Around what fraction of the class should have an 80% t-CI containing μ ?

80%

3. A 95% z-CI for μ based on a large sample selected with replacement from a population is given as [3.884, 3.9170].

$$\begin{aligned} \text{MOE} &= \frac{1}{2} \text{ diff of upper + lower CI} \\ &= \frac{3.9170 - 3.884}{2} = .033/2 = .0165 \end{aligned}$$

Interval for 68% confidence

$$\begin{aligned} \bar{x} &= \frac{3.884 + 3.9170}{2} \\ &= 3.9005 \end{aligned}$$

95% z-CI if instead the sampling is without replacement, population size $N = 1000$ and sample size $n = 100$.

$$\begin{aligned} &\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \\ &3.9005 \pm 1.96 \frac{s}{\sqrt{n}} \sqrt{\frac{1000-100}{1000-1}} \\ &\quad \quad \quad \underbrace{\hspace{2cm}}_{.0165} \\ &3.9005 \pm .0165 (.949/6) \end{aligned}$$

Class Data #1 Problem

Name	n	\bar{x}	s	$\frac{s}{\sqrt{n}}$	$1.96(1. \frac{s}{\sqrt{n}})$	68% of μ CI $\left[\bar{x} - (1) \frac{s}{\sqrt{n}}, \bar{x} + (1) \frac{s}{\sqrt{n}} \right]$	Cover $\mu=2?$
Emily	30	2.3	1.385	.253	.496	2.047 - 2.553	
Angie	30	1.77	1.36	.248	.489	1.522 - 2.018	✓
James	30	1.364	.7649	.1397	.2738	1.2263 - 1.5057	
Jorn	30	1.6	.9322	.1702	.334	1.4298 - 1.7702	
Tim	30	1.63	.999	.182	.337	1.448 - 1.812	
Tenika	30	1.9	1.155	.21	.4116	1.69 - 2.11	✓
Sarah	30	2.23	1.529	.279	.5471	1.9508 - 2.5091	✓
Jessica	30	1.9	3.78	.69	1.35	1.21 - 2.59	✓
Megan	30	1.7	1.32	.241	.4724	1.459 - 1.941	
Rachel	30	1.13	.819	.149	.29204	.981 - 1.279	
Tyrone	30	1.66	.95	.173	.061	1.48 - 1.83	
Takhyun	30	1.9	1.1552	0.2109	.04134	1.6891 - 2.1109	✓
Tiffany	30	2.166	1.39	.253	.497	1.913 - 2.419	✓

Class Data Problem #2

Name	n	\bar{x}	s	$\frac{s}{\sqrt{n}}$	$\bar{x} - t_{.1} \frac{s}{\sqrt{n}}, \bar{x} + t_{.1} \frac{s}{\sqrt{n}}$	$\mu = 5$ Cover?
Tim	5	5	2.65	1.59	2.11 - 7.89	✓
Angie	3	4.33	3.21	1.85	.8409 - 7.82	✓
James	3	.667	.8165	.471	.667 - .889	
Sarah	3	4.66	.577	.333	4.03 - 5.28	✓
Tiffany	3	4.66	1.15	.663	1.841 - 4.346	
Megan	3	5.3	3	1.73	2.937 - 7.873	✓
Jessica	3	4.33	.356	.617	3.166 - 5.493	✓
Jorn	3	4.667	.577	.333	4.028 - 5.296	✓
Tyrone	3	4.33	.153	.883	2.97 - 5.97	✓
Takhyun	3	5.617	.577	.333	5.039 - 6.295	
Tenika	3	5.33	2.57	1.45	3.476 - 7.1905	
Emily	3	5	1	.707	3.114 - 6.886	✓
Rachel	3	7.3	.845	.48786	6.5798 - 8.2201	✓

9/13 cover