Angie Davison



H.2

## 1. With replacement equal probability sample.

a. Use your block of random digits to select an equal probability with replacement sample of n = 30 pages from the textbook.

n=30

050 090 156 181 063 165 653 173
082 172 145 ~~013~~ 184 032 182 168
159 156 ~~051~~ 119 144 166 129 166
~~011~~ 182 154 ~~026~~ 059 200 062 046
166 149 187 ~~074~~ 126 033 041 201
074 123 125 081 036 092 166 053 136
147 ~~062~~ ~~068~~ 127 050 202 026
151 168 ~~036~~ 076 061 134 035
033 165 205
087

b. Score each sample page (above) with x = number of "KEY EXAMPLES" begun on that page.

11→2 74→0 74→0 172→0 156→1
30→1 81→0 159→0 156→1 145→0
50→1 82→0 166→1 182→0 154→1
51→1 83→0 123→0 149→1 187→1
62→1 90→1 147→1 125→0 181→1
68→1 26→0 151→0 168→1 119→0

c. Use your sample data from (b) to determine

$\bar{x}$ .567

s $\sqrt{\dfrac{(1-.567)^2 \times 15 + (2-.567)^2 + (0-.567)^2 \times 14}{29}} = .568$

68% z-CI for $\mu$ $.567 \pm (1)\dfrac{.568}{\sqrt{30}} = .463$ to $.671$ ~~-.001 to 1.135~~

typo

covers $\bar{x}$ as did the incorrect version

In class, we will calculate the (grand) average of all 18 students' values for sample mean. Although it is not the actual population mean $\mu$ it is likely to be very close. We'll see if around 68% of the classes' z-CI cover this grand average.

$MOE = 1.96 \dfrac{.568}{\sqrt{30}} = .203$

correct

## 2. Without replacement equal probability sample.

87 ~~99~~ 87

Set up the 2-digit correspondence 01↔page 1, ..., ~~99~~↔page ~~87~~, ... other pairs↔none.

Starting again at the beginning of your block of random digits peruse consecutive **pairs** of random digits, selecting random pages from only pages 1 through 87 of the textbook. Student #1 has digits beginning

876 904 531 090 491 806 584 704 102 709 ...

Grouping into consecutive **pairs**

87 69 04 53 10 90 49 18 06 58 47 04 10 27 ...

see that 04 occurs twice. If we want to sample with equal probability all we have to do is skip over any duplicate. That is, if we skip all repeats of 04 the remaining unseen page numbers the unseen page numbers remain equally likely to get into the sample.

So the student above obtains equal probability without replacement sample pages from the range 01 through 87 beginning as follows:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| digit pairs: | 87 | 69 | 04 | 53 | 10 | 90 | 49 | 18 | 06 | 58 | 47 | 04 | 10 | 27 | ... |
| page selected: | 87 | 69 | 4 | 53 | 10 | | 49 | 18 | 6 | 58 | 47 | | | 27 | ... |

a. Use your block of random digits to select an equal probability without replacement sample of n = 30 pages from the textbook.

| | | | |
|---|---|---|---|
| ✓83 | ✓70 | ✓67 | 23 |
| ✓26 | ✓03 | 48 | |
| ✓53 | ✓61 | 79 | |
| ✓33 | ✓59 | 14 | |
| ✓13 | ✓68 | 09 | |
| ✓19 | ✓11 | 51 | |
| ✓37 | ✓69 | 60 | |
| ✓05 | ✓47 | 74 | |
| ✓82 | ✓16 | 76 | |
| ✓65 | | 63 | |

1→87

b. Score each sample page (above) with x = number of "KEY EXAMPLES" begun on that page.

| | | | | | | |
|---|---|---|---|---|---|---|
| 03 0 | 19 1 | 26 0 | 48 1 | 61 1 | 60 0 | 82 0 |
| 05 0 | 11 2 | 23 1 | 47 2 | 63 0 | 68 1 | 83 0 |
| 09 1 | 16 0 | 33 0 | 51 1 | 65 2 | 74 0 | |
| 13 2 | 14 1 | 37 0 | 53 1 | 69 0 | 76 2 | |
| | | | 59 0 | 67 2 | 79 0 | |
| | | | | | 70 0 | |

c. Use your sample data from (b) to determine

$\bar{x}$ .7

$s$ $\sqrt{\dfrac{(1-.7)^2 \times 9 + (2-.7)^2 \times 6 + (0-.7)^2 \times 15}{29}} = .79$

$\bar{x} \pm z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

MOE $= 1.96 \times .144$

$= .282$

$\times \sqrt{\frac{87-30}{86}}$

$= .230$

68% z-CI for $\mu$ $.7 \pm 1 \frac{.79}{\sqrt{30}} \sqrt{\frac{87-30}{87-1}} = .583$ to $.817$

Correct

**Note:** In class, we will calculate the (grand) average of all 18 students' values for sample mean. Although it is not the actual population mean $\mu$ it is likely to be very close to $\mu$. We'll see if around 68% of the class' z-CI cover this grand average.

**3. Achieving a given precision by choosing a large sample.** Key 52 discusses how to choose sample size n in order to ensure that a 95% z-CI (in with replacement case) is not too wide. The form is:

$$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}$$

Unfortunately, you don't know what s will be until you get data. What to do? Take a preliminary sample. Estimate $\sigma$ by means of the sample standard deviation $s_{prelim}$ **of this preliminary sample.** If

$$1.96\, s_{prelim} / \sqrt{n_{prelim}} \leq W$$

then you are done since your regular z-CI already has the desired narrowness specified by W. Otherwise, solve for $n_{final}$ in

$$1.96 \frac{s_{prelim}}{\sqrt{n}} = W \text{ (W being any desired half-width)}$$

i.e. $n_{final} = (1.96\, s_{prelim} / W)^2$

Then continue sampling to the larger sample size $n_{final}$. Your 95% z-CI is then (approximately)

$\overline{x_{final}} \pm W$

which is what you wanted to achieve.

Be prepared to suffer large $n_{final}$ if you want W to be small (precise CI).

a. An experimenter wishes to estimate the mean failure pressure (psi) for a particular type of tire. They would like a 95% z-CI of the precision

$\overline{x} \pm 10$ psi

A preliminary sample of 100 tires produces a sample standard deviation of $s_{prelim}$ = $\boxed{42 \text{ psi.}}$

Determine the recommended total sample size

$$n_{final} = (1.96\, s_{prelim} / W)^2$$

$n_f = \left(\frac{1.96 \times 42}{10}\right)^2 = 67.76 \to \underline{68}$

Check that the recommended $n_{final}$ is not greater than 100. It means that the needed precision has already been achieved. Therefore, give the ordinary z-CI from the data already in hand if it is found that $\overline{x_{prelim}}$ = 192.32 psi.

$n_f < 100$ ✓

$68 < 100$

$192.32 \pm 10 = 182.32$ to $202.32$

b. With the data of (a), suppose we really desire a 95% z-CI of $\overline{x_{final}} \pm 1$ psi and are perhaps willing to employ the hybrid method. Determine

$$n_{final} = (1.96\, s_{prelim} / W)^2$$

$n_f = \left(\frac{1.96 \times 42}{1}\right)^2 = 6776.58 \to \underline{6,777}$

c. Suppose that you have continued to the recommended total sample size $n_{final}$ (you were allowed to include the initial 100 sample values) and your sample mean of all $n_{final}$ scores is $\overline{x_{final}}$ = 193.84.

Give the hybrid ~~z-CI~~

$\overline{x_{final}} \pm 1$ psi

~~192~~ 192.84 to 194.84

d. What margin of error will you quote for the hybrid method?

$\text{MOE} = 1.96 \frac{s}{\sqrt{n}}$

$1.96 \times \frac{42}{\sqrt{6777}} = .99997$

close to 1 as intended

may as well use 1

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①

|  | $n$ | $\bar{x}$ | $s$ | MOE<br>$1.96\frac{s}{\sqrt{n}}$ | LCL<br>$\bar{x}-(1)s/\sqrt{n}$ | UCL<br>$\bar{x}+(1)\frac{s}{\sqrt{n}}$ | cover $\bar{\bar{x}}$? |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  | 68% |  |  |
| Tim | 30 | .533 | .730 | .133 | .4 | .666 | ✓ |
| Angie | 30 | .567 | .568 | .203 | .463 ~~.061~~ | .67 ~~1.135~~ | ✓ |
| Talchyun | 30 | .633 | .8087 | .289 | .4857 | .7869 | ✓ |
| Omar | 30 | .133 | .1044 | .037 | .1109 | .1490 |  |
| Jorn | 30 | .6 | .8137 | .2912 | ~~-.0594~~ | .2377 |  |
| Sarah | 30 | .5 | .6297 | .225 | .3850 | .6149 | ✓ |
| Rachel | 30 | .83 | .90231 | .3228839 | ~~-.166526~~ | .9947385 | ✓ |
| James | 30 | .667 | .695 | .249 | .54 | .794 | ✓ |
| Emily | 30 | .967 | .808 | .289 | .819 | 1.115 |  |
| Ashleigh | 30 | .6 | 1.459 | .5221 | .3336 | .8664 | ✓ |
| AVG | $\bar{\bar{x}}$ = .603 |  | 7/10 ~ 68% |  |  |  |  |
|  |  |  |  |  |  |  |  |
| Tyrell | 30 | .4 | .632 | .27410 | .492 | 1.04 | ✓ |

(2)

$$\bar{x} \pm (1)\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

| Name | n | $\bar{x}$ | S | MOE | LCL | UCL | Cover $\mu$? |
|---|---|---|---|---|---|---|---|
| Tim | 30 | .733 | .740 | .216 | .624 | .842 | ✓ |
| Angie | 30 | .7 | .79 | .230 | .583 | .817 | |
| Takhyon | 30 | 1 | .8710 | .4545 | .8706 | 1.1294 | |
| OMar | 30 | .76 | .7409 | .264 | .624 | .895 | ✓ |
| Jorn | 30 | .9 | .7589 | .221 | .787 | 1.013 | ✓ |
| Sarah | 30 | .73 | .785 | .281 | .5807 | .8733 | ✓ |
| Rachel | 30 | 1.23 | 1.02945 | .29986 | -1.00 | 1.38299 | ✓ |
| James | 30 | .767 | .817 | .179 | .503 | .746 | |
| Emily | 30 | 1.067 | .943 | .274 | .729 | 1.009 | ✓ |
| Ashleigh | 30 | .633 | .7643 | | | | |
| Tyrell | 30 | .706 | .875 | .27410 | .492 | 1.04 | ✓ |

AVG $\bar{\bar{x}} = .838$ 7/11

(3)