
Final exam prep 8 - 13 - 10 Try these. Not handed in. Will go over in class. Report errors.

1. z-CI for p. An equal probability with replacement sample of 100 persons is selected from customers of a store by randomly alerting a clerk at checkout. Customers are offered a choice of one of two items A or B. It is found that 62 out of 100 choose item A over item B. The form of a 95% z-CI for the population fraction p of all customers who would choose item A over item B is

$$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

a. Evaluate the CI for the data given.

b. From the formula above identify

point estimate of p and its value for this data

MOE for \hat{p} and its value for this data

point estimate of sd of \hat{p} and its value for this data

c. $P(p \text{ is covered by } \boxed{\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}}) \sim$

d. Percentage of users of such CI whose CI covers $p \sim$
(assumes the users operate independently)

2. z-CI for μ with or without replacement. An equal probability **with** replacement sample of 40 pages is selected from a textbook of 469 pages. Each sample page is scrutinized to discover the number of errors "x" (what this means is carefully codified). It is found that these 40 x-scores have sample mean 0.73 and sample standard deviation $s = 1.51$.

a. Give mathematical form of a 95% z-CI for μ , the average number of errors per page in the entire book.

b. From the formula above identify

point estimate of μ and its value for this data

MOE for \bar{X} and its value for this data

point estimate of sd of \bar{X} and its value for this data

c. From rule or z-table (or your calculator) determine z for

68% CI

83% CI

d. How is the 95% z-CI to be modified if it is learned that the data was actually obtained from a without-replacement equal probability random sample? Write the explicit form of the CI and evaluate it.

3. Hybrid z-CI to achieve 95% z-CI of form $\bar{x}_{\text{final}} \pm 0.2$. In #2a, regard the sample of 40 as a **preliminary** with-replacement equal probability sample of $n_{\text{preliminary}} = 40$ (preliminary sample mean 0.73 and preliminary sample standard deviation $s_{\text{preliminary}} = 1.51$). We desire a 95% hybrid z-CI for μ of the form

$$\bar{x}_{\text{final}} \pm 0.2.$$

a. Evaluate the MOE for the preliminary 95% z-CI (same as in 2a). Does it already have at least the precision 0.2?

b. Equating $(1.96 \frac{s_{\text{preliminary}}}{\sqrt{n_{\text{final}}}}) = 0.2$ solve for the final sample size n_{final} needed by the hybrid z-CI. Is $n_{\text{final}} > 40$? If so, your answer to (a) must have been NO, you did not have the needed precision at $n = 40$.

c. Suppose we continue sampling up to n_{final} and find that $\bar{x}_{\text{final}} = 0.77$. Give the 95% hybrid z-CI for μ .

4. Match each of the CI below to their intended use at left (one "CI" is never used).

Hybrid z-CI

$$\bar{d} \pm z \frac{s_d}{\sqrt{n}}$$

For all $n > 1$, normal population only

$$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

For μ_x , large n, with-replacement sample)

$$\bar{x} \pm z \frac{s}{\sqrt{n}} \text{ FPC}$$

For μ without repl sample

$$\bar{x} \pm t_{\alpha, \text{df}} \frac{s}{\sqrt{n}} \text{ FPC}$$

Difference of means, unpaired, independent data

$$\bar{x}_{\text{final}} \pm W$$

Difference of means, paired data

$$(\bar{x} - \bar{y}) \pm z \sqrt{s_x^2 / n_x \oplus s_y^2 / n_y}$$

For population proportion

$$\bar{x} \pm t_{\alpha, \text{df}} \frac{s}{\sqrt{n}}$$

5. **A sample of $n = 5$ from a normal population.** Suppose the sample mean is 3.79 and the sample sd is $s = 2.45$. Determine

a. MOE

b. 95% CI for μ

c. In this setup, with samples from a normal population distribution, the hybrid method works for any small preliminary sample with $n > 1$. Using the appropriate replacement of 1.96 find the n required for a hybrid CI to achieve precision

$$\bar{x}_{\text{final}} \pm 0.2$$

Keep in mind, our preliminary sample of only $n = 5$ works because the population distribution is normal and we are using the correct replacement for 1.96.

d. If you do continue to the recommended sample size and find that $\bar{x}_{\text{final}} = 3.65$ what is the hybrid 95% CI?

e. Refer to (d). What is the MOE?