Final exam prep 8 - 13 - 10 Try these. Not handed in. Will go over in class. Report errors.

1. z-CI for p. An equal probability with replacement sample of 100 persons is selected from customers of a store by randomly alerting a clerk at checkout. Customers are offered a choice of one of two items A or B. It is found that 62 out of 100 choose item A over item B. The form of a 95% z-CI for the population fraction p of all customers who would choose item A over item B is

$$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

a. Evaluate the CI for the data given.

$\hat{p}$ = 62/100 = 0.62

CI for p is $.62 \pm 1.96 \frac{\sqrt{.62 \times .38}}{\sqrt{100}}$

b. From the formula above identify

point estimate of p and its value for this data

0.62 (estimate around 62% of customers favor A)

MOE for $\hat{p}$ and its value for this data

$$1.96 \frac{\sqrt{.62 \times .38}}{\sqrt{100}}$$

point estimate of sd of $\hat{p}$ and its value for this data

$$\frac{\sqrt{.62 \times .38}}{\sqrt{100}}$$

c. P(p is covered by $\boxed{\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}}$) ~ **0.95**

d. Percentage of users of such CI whose CI covers p ~ **95%**
(assumes the users operate independently)

2. z-CI for μ with or without replacement. An equal probability **with** replacement sample of 40 pages is selected from a textbook of 469 pages. Each sample page is scrutinized to discover the number of errors "x" (what this means is carefully codified). It is found that these 40 x-scores have sample mean 0.73 and sample standard deviation s = 1.51.

a. Give mathematical form of a 95% z-CI for μ, the average number of errors per page in the entire book.

$$\boxed{\bar{x} \pm z \frac{s}{\sqrt{n}}} = \boxed{\mathbf{0.73 \pm 1.96} \frac{1.51}{\sqrt{40}}}$$

b. From the formula above identify

point estimate of μ and its value for this data

$$\bar{x} = \mathbf{0.73}$$

MOE for $\bar{x}$ and its value for this data

$$\mathbf{1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{1.51}{\sqrt{40}}}$$

point estimate of sd of $\bar{x}$ and its value for this data

$$\mathbf{\frac{s}{\sqrt{n}} = \frac{1.51}{\sqrt{40}}}$$

c. From rule or z-table (or your calculator) determine z for

68% CI **1.00 rule of thumb**

83% CI **z-table entry for z = 0.83/2 = 0.415 ~ 1.3722**

d. How is the 95% z-CI to be modified if it is learned that the data was actually obtained from a without-replacement equal probability random sample? Write the explicit form of the CI and evaluate it. **With FPC =** $\mathbf{\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{469-40}{469-1}}}$ so $\boxed{\bar{x} \pm z \frac{s}{\sqrt{n}} \text{FPC}} = \boxed{0.73 \pm 1.96 \frac{1.51}{\sqrt{40}} \sqrt{\frac{469-40}{469-1}}}$

3. Hybrid z-CI to achieve 95% z-CI of form $\bar{x}_{final}$ ± 0.2. In #2a, regard the sample of 40 as a **preliminary** with-replacement equal probability sample of $n_{preliminary} = 40$ (preliminary sample mean 0.73 and preliminary sample standard deviation $s_{preliminary} = 1.51$). We desire a 95% hybrid z-CI for μ of the form

$$\bar{x}_{final} \pm 0.2.$$

a. Evaluate the MOE for the preliminary 95% z-CI (same as in 2a). Does it already have at least the precision 0.2?

$$\mathbf{1.96\ \frac{s}{\sqrt{n}} = 1.96\ \frac{1.51}{\sqrt{40}} \sim 0.467954 > 0.2}$$

(not meeting the desired precision)

b. Equating $(1.96\ \frac{s_{preliminary}}{\sqrt{n_{final}}}) = 0.2$ solve for the final sample size n_{final} needed by the hybrid z-CI. Is $n_{final} > 40$? If so, your answer to (a) must have been NO, you did not have the needed precision at n = 40.

$$n_{final} = (1.96\ \frac{s_{preliminary}}{0.2})^2 = (1.96\ \frac{1.51}{0.2})^2 \sim \mathbf{219}$$

c. Suppose we continue sampling up to n_{final} and find that $\bar{x}_{final} = 0.77$. Give the 95% hybrid z-CI for μ.

~ 0.77 ± 0.2 as desired

4. Match each of the CI below to their intended use at left (one "CI" is never used).

5 Hybrid z-CI	$\bar{d} \pm z \frac{s_d}{\sqrt{n}}$	**1**
7 For all n > 1, normal population only	$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$	**2**
For μ_x, large n, with-replacement sample)	$\bar{x} \pm z \frac{s}{\sqrt{n}}$ FPC	**3**
3 For μ without repl sample	$\bar{x} \pm t_{\alpha,\mathrm{df}} \frac{s}{\sqrt{n}}$ FPC	**4**
6 Difference of means, unpaired, independent data	$\overline{x_{\mathrm{final}}} \pm W$	**5**
1 Difference of means, paired data	$(\bar{x} - \bar{y}) \pm z \sqrt{s_x^2 / n_x \oplus s_y^2 / n_y}$	**6**
2 For population proportion	$\bar{x} \pm t_{\alpha,\mathrm{df}} \frac{s}{\sqrt{n}}$	**7**

5. A sample of n = 5 from a normal population. Suppose the sample mean is 3.79 and the sample sd is s = 2.45. Determine

a. MOE

$$t_{\alpha,\,df}\,\frac{s}{\sqrt{n}} = t_{0.025,\,5-1}\,\frac{2.45}{\sqrt{5}} \sim 2.777\,\frac{2.45}{\sqrt{5}}$$

b. 95% CI for μ

$$\bar{x} \pm t_{\alpha,\,df}\,\frac{s}{\sqrt{n}} = 3.79 \pm 2.777\,\frac{2.45}{\sqrt{5}}$$

c. In this setup, with samples from a **normal population** distribution, the hybrid method works for **any** preliminary sample with n > 1. Using the appropriate replacement of 1.96 find the n required for a hybrid CI to achieve precision

$\bar{x}_{\text{final}} \pm 0.2$

Keep in mind, our preliminary sample of only n = 5 works because the population distribution is normal and we are using the correct replacement for 1.96.

$$n_{\text{final}} = \left(2.777\,\frac{s_{\text{preliminary}}}{0.2}\right)^2 = \left(2.777\,\frac{2.45}{0.2}\right)^2 \sim 1158 \text{ (round up)}$$

d. If you do continue to the recommended sample size and find that $\bar{x}_{\text{final}} = 3.65$ what is the hybrid 95% CI?

~ 3.65 ± 0.2

e. Refer to (d). What is the MOE?

~ 0.2 as desired (but it sure costs a lot of sampling)