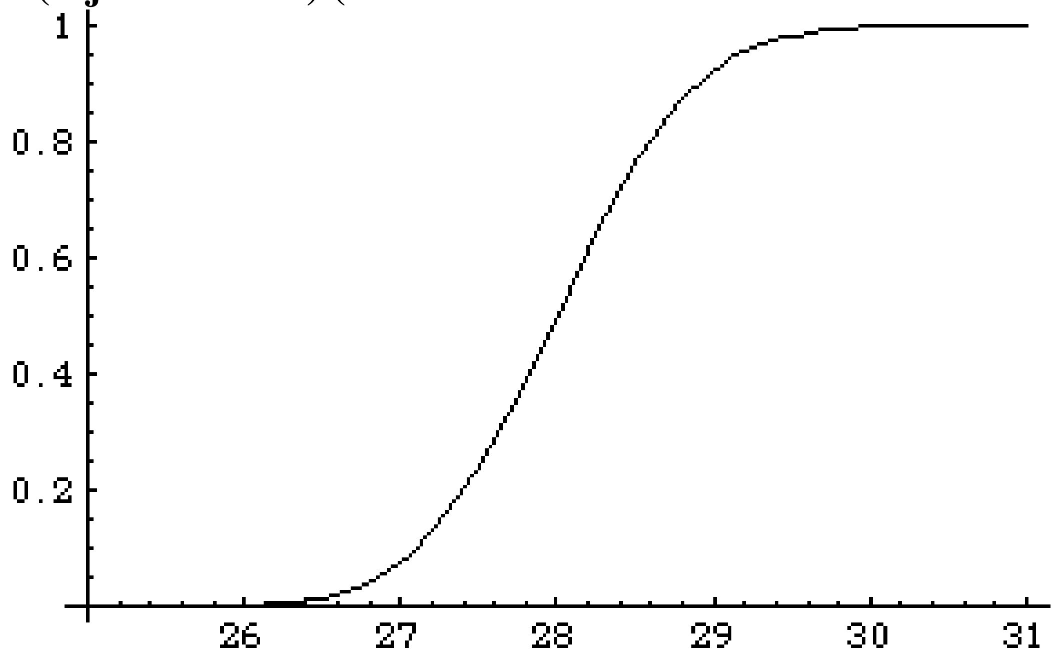
1. **NOTE: THIS ASSIGNMENT DEALS WITH THE POWER CURVE AND METHODS OF DESIGNING A "HYBRID TEST." INFORMATION FROM A PRELIMINARY SAMPLE DETERMINES A NEEDED SAMPLE SIZE IN ORDER THAT OUR HYBRID TEST ACHIEVE A GIVEN ALPHA AND A PARTICULAR BETA.** For a particular test (not specified) here is a plot of P(reject H0 | mu) as mu varies between 25 and 31. I will tell you that the boundary between H0 and H1 for this test is mu0 = 27.

P(reject H0 | mu) (which is IDEALLY 0 on H0 and 1 on H1)



H0 ->|<- H1 mu -->

a. Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the curve.

ANS. Alpha is the height above the boundary value mu_0 = 27, which is (by inspection of this curve) alpha ~ 0.15.

b. Give a fairly accurate numerical value for the power and beta at mu1 = 28. Illustrate what you are doing in the curve.

ANS. power is the height at 28, which is around 0.47. So beta (the chance of failing to reject whn mu is 28) is ~ 0.53.

c. Identify the null and alternative hypotheses.

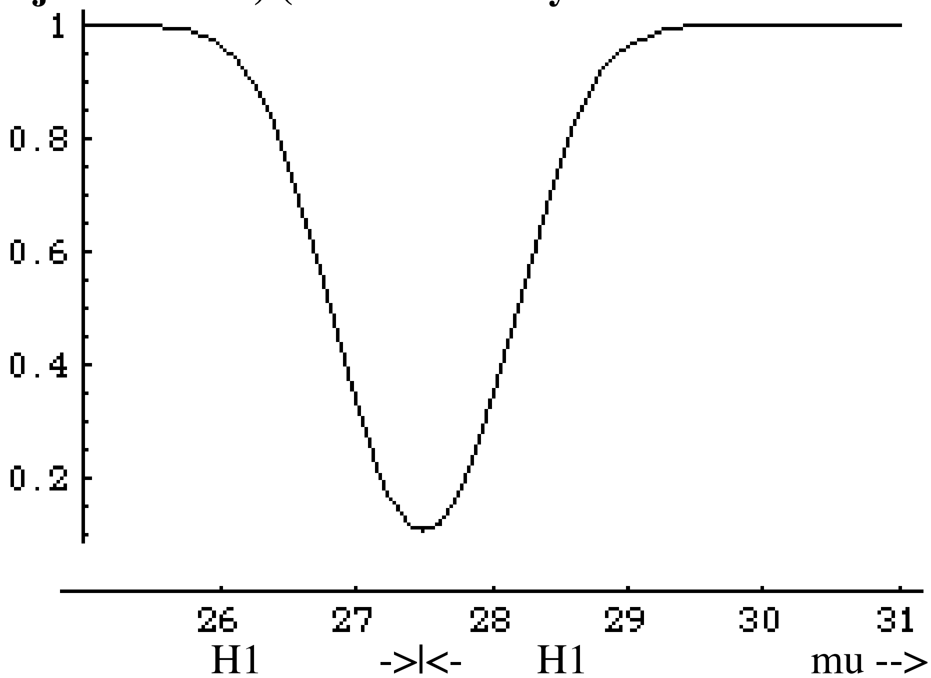
ANS. H0 = is where reject probability is low so it is {mu less or equal 27} ; H1 = {mu > 27} where reject probability is high.

d. In the above plot, overlay another curve representingP(reject H0 | mu) for a BETTER test of these hypotheses with the SAME ALPHA.

ANS. Overlay a curve that is higher everywhere on H1 and lower everywhere on H0 except at $\mathbf{mu_0}$ where it is the same alpha = 0.1 (i.e. having better performance for the same alpha).

2. For another test the plot of P(reject H0 | mu) as mu varies between 25 and 31 has a shape different from (1).

P(reject H0 | mu) (which is ideally 0 on H0 and 1 on H1)



a. Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the curve.

ANS. This is evidently a two sided test and alpha is the height of curve at $\mathbf{mu_0}$ = 27.5 which is alpha ~ 0.12 (just be close).

b. Give a fairly accurate numerical value for power and beta at mu1 = 28.5. Illustrate what you are doing in the curve.

ANS. Power at mu1 = 28.5 is the height there. It is around 0.78 (just be close). Beta = 1 - power ~ 0.22.

c. Identify the null and alternative hypotheses.

ANS. H0 = {mu = 27.5}.
H1 = {mu is not equal to 27.5}

d. Overlay on the above plot a curve representing a BETTER test for the SAME value of alpha.

ANS. Overlay a curve everywhere higher than the given one except having the same height at mu_0 = 27.5 (preserving alpha).

3. Calculate sample standard deviation "s" for the data {4, 5, 11, 12}.

ANS. The sample mean is xBAR = (4+5+11+12)/4 = 8.
s = root([$(4\text{-}8)^2 + (5\text{-}8)^2 + (11\text{-}8)^2 + (12\text{-}8)^2$] / [4-1])
= root(50 / 3).

4. a. A particular one-sided z-test of an H0 to the left of H1, and having alpha = 0.05, rejects H0 if the test statistic EXCEEDS which (tabled) value?

ANS. z(alpha) = z(0.05) = 1.645.

b. Consider a TWO-sided t-test (**NOT z-test**) of H0: mu = 16 ounces, with n = 10 and alpha = 0.1. Evaluate the **t**-threshold for rejection (applicable if the population is normal).

ANS. DF = 9 and t(alpha / 2) = t(0.05) = 1.833.

c. Refer to (4b). If the test statistic turns out to equal -1.9 what action is taken by the test?

ANS. Two sided test rejects H0 if ABSOLUTE VALUE of test statistic exceeds threshold 1.833. So it rejects H0.

d. Refer to (4b). An initial sample of n0 = 10 has sample standard deviation s0 = 6.1. What is the recommended total sample size n to achieve alpha = 0.1 and, at mu = 16.5, beta = 0.025? Refer to page 314. Remember, the test is TWO-sided.

ANS. For the two sided test t0 = t(alpha / 2) = 1.833 but t1 = t(beta) = t(0.025) = 2.262, <u>in all cases, even for the two sided test</u>. From the formula of page 314 (we use s from a

preliminary sample in place of sigma which is typically not known):
n = ((|t0| + |t1|) s0 / (mu0 – mu1))^2
= ((1.833 + 2.262) 6.1 / (16 – 16.5))^2 = 2496

e. Refer to (4d). If the sample mean of all n = 2496 is xBAR = 16.23 what is the value of the Hybrid Test Statistic? What is the action taken by the test?

ANS. The Hybrid Test Statistic (16.23 – 16) / (6.1 / root(2496)) = 1.884. Notice that the HYBRID TEST employs xBAR from all n = 2496, and also employs root(2496), but it <u>sticks with s0 = 6.1 from the initial sample</u> <u>of 10.</u> This two-sided test rejects H0 if the ABSOLUTE VALUE of the HYBRID test statistic exceeds 1.833 (determined from the initial sample size 10). So it does reject H0 since |1.884| > 1.833.

5. Typically, p = 0.50 of e-customers shop after 6 p.m. EST. We decide to test the null hypothesis H0: p (< or =) 0.5 versus H1: p > 0.5, with alpha = 0.05.

a. An initial sample of 100 e-customers finds 54 who shop after 6 p.m. Determine the numerical value of the test statistic based upon pHAT (which you must identify). Use the form of the test statistic which estimates sd by root(p0 q0). Reduce the test statistic to a number and be sure to say if it is positive or negative.

ANS. pHAT = 54/100 = 0.54. Test statistic (pHAT – 0.5) / (root(0.5 0.5) / root(100)) = (0.54 – 0.5) / (0.5 / 10) = .04 (20) = 0.8 (positive).

b. Determine the rejection threshold of a z-test for (5a) and state which action, reject H0 or fail to reject H0, is taken.

ANS. For this one sided test we reject H0 for values of the test statistic greater than the z for which P(Z > z) = alpha = 0.05. That value is z = 1.645 (see DF = infinity in t-table). Since TS = 0.8 does not exceed z = 1.645 we fail to reject H0.

c. We've failed to reject in (b) but maybe it is really true that pHAT > 0.5 and we simply did not gather enough data to gain the needed precision to reject H0. What n do we need in order to utilize a Hybrid Test for which alpha = 0.05 but also, for p = 0.53, beta = 0.08?

ANS. See pg. 319. z0 = 1.645 and z1 = 1.41 (since P(Z > 1.41) ~ 0.08). Continue sampling to

n = [(|z0| root(p0 q0) + |z1| root(p1 q1)) / (p0-p1)] ^{2}

= [(1.645 root(0.5 0.5) + 1.41 root(0.53 0.47)) / (.5 - .53)] ^{2}

= 2589 (round up)

d. If we do continue sampling to n = 2589 as recommended in (c) and if we find 1338/2589 customers who e-transact after 6 p.m. what is the Hybrid Test Statistic and what action is taken?

ANS. The Hybrid TS (HTS) is

(pHATfinal – p0)/root(p0 q0 / nFINAL)

and the test rejects "H0:p < = 0.5" if this HTS > 1.645 (the same value of z used by the initial test).

HTS = (1338/2589 – 0.5)/root(0.5 0.5 / 2589) = 1.71

Since this exceeds 1.645 the test based on the more stringent criteria and consequently larger sample, rejects H0.

e. Sketch the general appearance ofP(rej H0 | mu) for this one sided test having alpha = 0.05 and, at p = 0.53, beta = 0.08. Clearly identify alpha, beta, p0, p1 null and alternative hypothesis as recognizable entities in your sketch.

ANS. The general shape of (1).

6. A test statistic for a **z-test** evaluates (from the data) to 3.14.

a. If the hypothesis is H0: p = 0.4, and the alternative is H1: p is not 0.4, what is the numerical value of pSIG?

ANS. For a two sided test pSIG is

$P(|Z| > 3.14) = 2\ P(Z > 3.14) = 2\ (0.5 - 0.4992)$

Always, pSIG is the probability of data more disagreeable with H0 than is our data SAM.

b. Refer to (6a). IfpSIG = 0.22 and alpha = 0.32 what action is taken by the z-test and why?

ANS. Always, the rule is to reject H0 if pSIG < alpha. In this case we reject H0.

ASSIGNMENT BELOW!

Assignment due 10-26-06 in recitation.

1. A test of H0: mean income < 29 versus H1: mean income > 29 has alpha = 0.05 and, for mu = 29.5, has beta = 0.1.

a. Sketch the general appearance of a plot of P(reject H0 | mu) vs mu. Clearly indicate alpha and beta, 29, 29.5 as recognizable entities in your sketch, which should "blow up" for detailed view the vicinity of the interval [28, 31].

b. Overlay on (a) the plot of P(reject H0 | mu) vs mu for the IDEAL case in which we could census the entire population (i.e. actually know if H0 is true or not).

c. Overlay on (a) the likely shape of a better test (larger beta) also having alpha = 0.05 but based on a larger sample size.

d. Assume a preliminary sample of 100 is available from which the sample sd is s = 0.61. Give the sample size n needed for a z-based Hybrid Test with alpha = 0.05 and, at mu = 29.5, beta = 0.1. Clearly identify z0, z1, mu0, mu1.

e. If the sample of larger size (d) is obtained and we find that the overall sample mean for this larger sample is xBARfinal = 29.03, what is the value of the Hybrid Test Statistic and what action is taken by the test?

2. A test of H0: mean income = 29 versus H1: mean incomeis not 29 has alpha = 0.05 and, for mu = 29.5, has beta = 0.1.

a. Sketch the general appearance of a plot of P(reject H0 | mu) vs mu. Clearly indicate alpha and beta, 29, 29.5 as recognizable entities in your sketch, which should “blow up” for detailed view the vicinity of the interval [28, 31].

b. Overlay on (a) the plot of P(reject H0 | mu) vs mu for the IDEAL case in which we could census the entire population (i.e. actually know if H0 is true or not).

c. Overlay on (a) the likely shape of a better test (larger beta) also having alpha = 0.05 but based on a larger sample size.

d. Assume a preliminary sample of 100 is available from which the sample sd is s = 6.1. Give the sample size n needed for a z-based Hybrid Test with alpha = 0.05 and, at mu = 29.5, beta = 0.1. Clearly identify z0, z1, mu0, mu1.

e. If the sample of larger size (d) is obtained and we find that the overall sample mean for this larger sample is xBARfinal = 29.03, what is the value of the Hybrid Test Statistic and what action is taken by the test?