For questions 1 thru 3, R, G denote the numbers thrown by a red die and a green die respectively.

1. List all the sample points and by counting the numbers of points in each of the following events give their probabilities (|R - G| denotes the absolute value of the difference). This is the distribution of the random variable \( X = |R - G| \).

\[
P\{|R - G| = 0\} = \frac{10}{36} \quad \text{(see sol’n on next page)} \\
P\{|R - G| = 1\} = \frac{10}{36} \\
P\{|R - G| = 2\} = \\
P\{|R - G| = 3\} = \\
P\{|R - G| = 4\} = \\
P\{|R - G| = 5\} = __________
\]

**total = 1 (why?)**
1. cont.

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
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</tbody>
</table>

 ✓ = \{|R-G| = 1\}

ans.

\[ P( |R-G| = 1 ) = \frac{10}{36} \]

2. From your answer to problem #1, plot the discrete probability density for the random variable (r.v.) \( X = |R - G| \).
3. By eye, in your answer to problem #2, attempt to identify the mean and standard deviation of the r.v. $X$.
   \[
   \text{mean} = \text{balance} \approx \\
   \text{s.d. (compare bell curve)} \approx 
   \]

4. Refer to #1, compute the **expected value** $E X$ of r.v. $X$. Work in a table.

   \[
   \begin{array}{ccc}
   x & P(X = x) & x \cdot P(X = x) \\
   \hline
   0 & & \\
   1 & & \\
   2 & & \\
   3 & & \\
   4 & & \\
   5 & & \\
   \text{totals} & 1 & E X = \sum x \cdot P(X=x) = \\
   \end{array}
   \]
5. Refer to #1, compute the variance of r.v. X directly.

\[
\begin{array}{cccc}
  x & P(X = x) & (x - EX)^2 P(X = x) \\
  0 & & \\
  1 & & \\
  2 & & \\
  3 & & \\
  4 & & \\
  5 & & \\
\end{array}
\]

\[
\text{Var}(X) = E (X - EX)^2 = \sum (x - EX)^2 P(X = x) =
\]

6. Refer to #1, compute variance of r.v. X using the short form.

\[
\begin{array}{cccc}
  x & P(X = x) & x^2 P(X = x) \\
  0 & & \\
  1 & & \\
  2 & & \\
  3 & & \\
  4 & & \\
  5 & & \\
\end{array}
\]

\[
EX^2 \text{ (this is not the variance!)} =
\]
7. \( P(\text{OIL}) = 0.3, \ P(\text{+ | OIL}) = 0.8, \ P(\text{+ | no OIL}) = 0.1. \)

Make a complete tree diagram.

8. Refer to problem #7. Suppose the cost to drill for oil is 50, and the cost of testing is 10. If oil is there the gross return (not considering drilling or testing costs) is 400 (zero if no oil is there). Fill out all four branches of your tree with the NET PROFIT (it may be in some contingencies be negative).
9. Refer to problem # 8. Compute the expected NET PROFIT if we test first, then drill only if the test is positive.

10. Refer to problem # 8. Compute the expected NET PROFIT if we do not test but just drill. Is the test worth it?
11. Give the distribution, mean, variance of the r.v. \( X = \) number of girls in four births (\( p = 0.5 \) for a girl, independent births).

12. If random variable \( W \) has \( E W = 4 \) and random variable \( R \) has \( E R = 7 \) then \( E(2 R = W) = \)

13. If random variable \( W \) has variance 12 and random variable \( R \) has variance 19 and \( W, R \) are INDEPENDENT then the variance of random variable \( (2R - W) = \)