QUIZ 2 PREP
IMPORTANT: Quiz 2 is comprehensive for Ch. 3 thru pg. 142. So it will also cover material from Project 4.

1. Evaluate the probability \( P(X = 3) \) where r.v. \( X \) is the "number of heads in 7 tosses of a fair coin." Do this using the formula (3-12) of your textbook.

   a) 14/128  b) 28/128  c) 69/128  d) 49/128  e) 35/128

2. Refer to problem 1. From the table of cumulative binomial probabilities TABLE 1 of your textbook find

   \[ P( X \leq 3) = \]

   \[ P(X \leq 2) = \]

   from which you find \( P(X=3) = P( X \leq 3) - P(X \leq 2) \). Compare with your answer to problem 1.

   a) 0.109  b) 0.218  c) 0.539  d) 0.382  e) 0.273

3. From (3-13) of your textbook calculate the expected value \( E X \) where r.v. \( X \) is the "number of defective castings in a shipment of 200 castings if each casting is defective with probability 0.35 and defects are independent."

   a) 35  b) 17.5  c) 100  d) 70  e) 0.35

4. From (3-14) of your textbook what is the variance \( \text{Var} X \) where r.v. \( X \) is as in question 3.

   a) 51.4  b) 45.5  c) 28.2  d) 64.4  e) 36.1
5. Refer to problems 3, 4 for $E X$ and $Var X$. Was it necessary to have independence of castings to justify your answers?

a) yes, yes  b) yes, no  c) no, yes  d) no, no

6. Evaluate the binomial probability that $X = 6$ where r.v. $X$ is the "number of lightning hits when there are a fixed number 80,000,000 of pre-hits and each pre-hit results in a hit with probability $1/10,000,000". Assume that hits are independent. Your answer will be difficult to evaluate numerically since it is the product of three terms

\[
\frac{80,000,000 \choose 6}{(1/10,000,000)^6} (9,999,999/10,000,000)^{80,000,000-6}
\]

(very large indeed, see page 119)

(very small indeed)

(intermediate sized number)

Poisson approximation of binomial is given next.

7. The Poisson approximation applies to problem 6 since the number of trials is very large (80,000,000) and the probability of success (hit) at each trial is very small (1/10,000,000). SO THE DISTRIBUTION OF THE NUMBER OF HITS DEPENDS APPROXIMATELY (ONLY) ON THE EXPECTED NUMBER OF HITS AND IS GIVEN APPROXIMATELY BY THE POISSON. What is this expected number of hits?

a) 1,000,000  b) 122  c) 8  d) 1/8  e) 8,000

8. Evaluate $P(X = 6)$ in problem 6 using the Poisson approximation of the binomial and using the formula on page 126 of your textbook.

a) $e^{-8} 8^6 / 6!$  b) $e^{-8} 6^8 / 6!$  c) $e^{-6} 6^8 / 8!$  d) $e^{-8} 6^8 / 8!$

9. Evaluate $P(X=6)$ in problem 6 using the tables of the Poisson Table 12 of your textbook.

a) 0.1221  b) 0.7826  c) 0.1032  d) 0.0139
10. We expect an average of 7.8 work stoppages per week and the distribution of r.v. X equal to "the number of work stoppages next week" is thought to be Poisson. Using the formula of the Poisson give $P(X = 5)$.

a) 0.198  
   b) 0.226  
   c) 0.201  
   d) 0.0669  
   e) 0.0986

11. Solve problem 10 using instead Table 12.

a) 0.198  
   b) 0.226  
   c) 0.201  
   d) 0.0669  
   e) 0.0986

A key point to remember: The Poisson can arise as an approximation of the binomial when $n$ is large and $p$ is small. But it can also arise in other ways, as a distribution in its own right. The Poisson distribution is entirely determined by its mean!