PREP FOR EXAM 3.
A KEY WILL BE POSTED.
QUESTIONS WILL BE ANSWERED IN LECTURE ON MONDAY, 29.

This prep

1-3. **Sampling distribution of $\bar{X}$ around the population mean $\mu_x$.**

1. Teachers are scored $x =$ number of years of advanced education. The population mean is $\mu_x = 7.2$ with population standard deviation $\sigma_x = 1.8$. For samples of $n = 40$ with-replacement
   a. The mean of the list of all possible sample means $\bar{x} = $

   b. The standard deviation of the list of all possible sample means $\bar{x} = $

   c. Sketch the approximate distribution of the list of all possible sample means $\bar{x}$ identifying all crucial features.

2. Same as problem 1, but suppose that sampling is WITHOUT replacement and the number of teachers in the population is 42,993.
   a. The mean of the list of all possible sample means $\bar{x} = $

   b. The standard deviation of the list of all possible sample means $\bar{x} = $

   c. Sketch the approximate distribution of the list of all possible sample means $\bar{x}$ identifying all crucial features.

3. Accounts are scored $x =$ amount outstanding. We sample $n = 41$ accounts with-replacement, finding sample mean $\bar{x} = 243.78$ and sample standard deviation $s = 176.77$.

   a. Our estimate of the population mean $=$
b. Our estimate of the population standard deviation =

c. Our estimate of the standard deviation of the list of all possible sample means =

4-5. 0-1 data. Sampling distribution of $\hat{p}$ around the population proportion (mean) $\mu_x = p$.

4. A population of 340 persons has scores $x = 1$ (favor proposal), $x = 0$ (opposed). If the proportion $p = \frac{\text{# in favor}}{N} = 0.77$,
   a. The population mean is $\mu_x =$
   
      b. The population standard deviation is $\sigma_x =$
   
      c. The s.d. $\sigma_{\bar{x}}$ of the list of all possible with-replacement $n = 20$ sample means =
   
      d. Sketch the approximate (normal) distribution of the list of all possible sample means $\bar{x}$ for with-replacement sample size $n = 130$. Label all key features.
   
      e. Sketch the approximate (normal) distribution of the list of all possible sample means $\bar{x}$ for WITHOUT-replacement sample size $n = 130$ if the population size is 340. Label all key features.

5. A with-replacement sample of $n = 60$ accounts has exactly 13 that are in arrears (behind in payments). Score $x = 1$ (in arrears), $x = 0$ (paid up). Let $p = (unknown)$ population fraction in arrears.
   a. Your estimate of the population proportion $p =$
b. Your estimate of the population standard deviation $\sigma_x =$ 

c. Your estimate of $\sigma_{\bar{X}} =$


6. A measure $x$ of customer satisfaction has been brought under statistical control. It has population mean 7.6 and standard deviation 0.86.

   a. Sketch the approximate distribution of $x$ with all important features.

   b. The standard score of $x = 7.92$ is

   c. $P(0 < X < 7.92 ) =$

   d. $P(7.92 < X ) =$

   e. $P(6.88 < X < 7.92) =$

   f. Find the 77th percentile of $Z$ (use closest table entry).

   g. Find the 77th percentile of customer satisfaction.

   e. Sketch the approx distribution of $y = 5 \times x + 3$ with all important features.

7-8. CLT probability calculations for sum or average of a sample.

7. (Refer to problem 1-c). Teachers are scored $x =$ number of years of advanced education. The population mean is $\mu_x = 7.2$ with population standard deviation $\sigma_x = 1.8$. For samples of $n = 40$ with-replacement

   a. In order to approximate $P( \bar{X} < 7.36) \text{ a number } z \text{ must be entered}$
to the table of the standard normal. What is that number z?

b. Evaluate the CLT approximation of \( P(0 < \bar{X} < 7.36) \).
c. Evaluate the CLT approximation of \( P(\bar{X} < 7.36) \).
d. Evaluate the CLT approximation of \( P(40 \bar{X} < 294.4) \).

8. (Refer to problem 4-d). A population has scores \( x = 1 \) (favor proposal), \( x = 0 \) (opposed). If the proportion \( p = \frac{\# \text{ in favor}}{N} = 0.77 \)
a. Denote by \( \hat{p} \) the SAMPLE proportion favoring the proposal for a sample of 130 selected WITH-replacement. In order to evaluate the CLT approximation of \( P(\hat{p} < 0.76) \) a z-score must be entered into the standard normal table. What is that z-score?
b. Evaluate the CLT approximation of \( P(\hat{p} < 0.76) \).
c. Evaluate the CLT approximation of \( P(\text{number favoring the proposal in a sample of 130 selected WITH-replacement is} < 103) \).
d. (Refer to problem 4-e). For a sample of 130 selected WITHOUT-replacement, evaluate the CLT approximation of \( P(\hat{p} < 0.76) \).


9. An estimator \( \hat{\theta} \) of a parameter \( \theta \) may have any of several properties.
a. Define an UNBIASED estimator.
b. Define an asymptotically UNBIASED estimator.
c. Express the typical size of the (random) sampling error \( \hat{\theta} - \theta \) in terms of some "order of sample size n." Do the same for the typical size of the bias \( E \hat{\theta} - \theta \). Typical means in many, but not all, standard statistical applications.
d. Define a sufficient statistic \( \hat{\theta} \). The answer is not the definition given in your textbook.
e. Define an asymptotically efficient estimator \( \hat{\theta} \) of a parameter \( \theta \).