The first 3 questions refer to one of the distributions below, depending upon your SLD:

<table>
<thead>
<tr>
<th>x</th>
<th>p(x)</th>
<th>x</th>
<th>p(x)</th>
<th>x</th>
<th>p(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.2</td>
<td>2</td>
<td>0.2</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>4</td>
<td>0.1</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>6</td>
<td>0.7</td>
<td>7</td>
<td>0.7</td>
</tr>
</tbody>
</table>

[2 Pts.] For your SLD, calculate \( \text{Var} X \) by any method. Show your method below, reducing the answer to a number.

\[
\begin{array}{ccccc}
    & x & p(x) & x^2 & p(x) & (x - E X)^2 & p(x) \\
3 & 3 & 0.2 & 1.8 & 0.6 & (3 - 4.5)^2 & 0.2 \\
4 & 4 & 0.1 & 1.6 & 0.4 & (4 - 4.5)^2 & 0.1 \\
5 & 5 & 0.7 & 17.5 & 3.5 & (5 - 4.5)^2 & 0.7 \\
\end{array}
\]

\[
\begin{align*}
    & E X = 4.5 & E X^2 = 20.9 & \text{Var} X = 0.65 \\
\end{align*}
\]

Note: \( \text{Var} X = E X^2 - (E X)^2 = 20.9 - 20.25 = 0.65 \) (short)
\( \text{Var} X = E (X - E X)^2 = 0.65 \) (definition)

Note: s.d. \( X = \sqrt{\text{Var} X} = \sqrt{0.65} \).

[1 Pt.] Refer to your X above. Depending upon your SLD, calculate \( \text{Var}(6X - 3) \text{ Var}(3X + 9) \text{ Var}(2X + 13) \)
Show your reasoning and reduce your answer to a number.

\[
\text{Var (6X - 3)} = \text{Var (6X)} = 36 \text{ Var X} = 36 (0.65)
\]

[1 Pt.] Refer to your X above. Depending upon your SLD,

\begin{align*}
\text{SLD 0, 1, 2} & & \text{SLD 3, 4, 5} & & \text{SLD 6, 7, 8, 9} \\
E(X) & = 3.6 & = 1.8 & = 7.2 \\
\text{Var X} & = 5.6 & = 27.2 & = 17.6
\end{align*}

\[
E (6X - 3) = 6 \ E X - 3 = 6 (4.5) - 3
\]

The next 4 questions refer to independent r.v. X and Y having expectations & variances specified (for your SLD)

\begin{align*}
\text{SLD 0, 1, 2} & & \text{SLD 3, 4, 5} & & \text{SLD 6, 7, 8, 9} \\
E(Y) & = 2.8 & = 5.6 & = 1.4 \\
\text{Var Y} & = 8.6 & = 3.2 & = 5.6
\end{align*}

[1 Pt.] Calculate E(X Y). Show your reasoning and reduce your answer to a number.

\[
E XY = (E X) (E Y) = (3.6) (2.8) \text{ for independent r.v. X, Y}
\]

[2 Pts.] Var(0.25 X + 0.5 Y). Show your reasoning and reduce your answer to a number.

\[
\text{Var(0.25 X + 0.5 Y)} \\
= \text{Var(0.25 X)} + \text{Var(0.5 X)} \text{ (by independence of X, Y)}
\]
\[(0.25)^2 \text{ Var } X + (0.5)^2 \text{ Var } Y = (0.25)^2 5.6 + (0.5)^2 8.6\]

[1 Pt.] Var(Y - X). Show your reasoning and reduce your answer to a number.

\[
\text{Var}(Y - X) = \text{Var}(Y) + \text{Var}(-X) \text{ by independence} \\
= \text{Var}(Y) + \text{Var}(X) \text{ since } \text{Var}(-X) = (-1)^2 \text{ Var}(X) \\
= 5.6 + 8.6
\]

[1 Bonus Pt.] \(\sigma_{X+Y}\). Show your reasoning and reduce your answer to a number.

\[
\sigma_{X+Y} = \sqrt{\text{Var}(X + Y)} = \sqrt{\text{Var}(X) + \text{Var}(Y)} \\
= \sqrt{5.6 + 8.6}
\]

The next 5 questions concern particular distributions.

[1 Pt.] Lifetimes of bearings are exponentially distributed
for SLD 0, 1, 2 for SLD 3, 4, 5 for SLD 6, 7, 8, 9

\[
\begin{align*}
\text{E } X & \quad 50.4 \text{ years} & \quad 31.2 \text{ years} & \quad 37.6 \text{ years} \\
\text{calculate} & \quad P(X > 25.2) & \quad P(X > 15.6) & \quad P(X > 18.8)
\end{align*}
\]

Write the correct expression and evaluate it completely.

For exponential having mean \(\mu\) and every value \(x > 0\)

\[
P(X > x) = e^{-x/\mu} \text{ so } P(X > 25.2) = e^{-25.2/50.4} = 0.6065.
\]

[1 Bonus Pt.] The lifetime of a type of part is exponentially
distributed. Calculate $P(\text{part fails after its average lifetime})$. Reduce your answer to a number.

$$P(X > \mu) = e^{-\mu/\mu} = e^{-1} = 0.367879.$$ 

On average, only 37% of parts live beyond average lifetime, unlike the normal which has 50% beyond the mean.

[1 Pt.] Each time a person is served at an automatic teller there is constant probability $p$ that assistance will be required. These requests for assistance are independent.

<table>
<thead>
<tr>
<th>SLD</th>
<th>p</th>
<th># served</th>
<th>p(x) for x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1, 2</td>
<td>0.23</td>
<td>198</td>
<td>48</td>
</tr>
<tr>
<td>3, 4, 5</td>
<td>0.31</td>
<td>231</td>
<td>70</td>
</tr>
<tr>
<td>6, 7, 8, 9</td>
<td>0.29</td>
<td>654</td>
<td>200</td>
</tr>
</tbody>
</table>

Calculate $P(X = x) = p(x)$, the probability that the specified number $x$ out of the number of customers served will need assistance. Leave factorials and powers unevaluated.

$$p(48) = \frac{198!}{48!150!} 0.23^{48} 0.77^{150} \text{ (Binomial distribution)}$$

[1 Pt.] The number $X$ of food complaints experienced in one year at a restaurant is Poisson

<table>
<thead>
<tr>
<th>SLD</th>
<th>$E X$</th>
<th>$p(255)$</th>
<th>$p(312)$</th>
<th>$p(615)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1, 2</td>
<td>259.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3, 4, 5</td>
<td>314.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6, 7, 8, 9</td>
<td>612.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate $p(255)$, $p(312)$, and $p(615)$.

Leave powers, etc., unevaluated in your answer.

$$p(255) = e^{-259.4} 259.4^{255} / 255!$$
[1 Pt.] Sketch the normal (bell) approximation of your Poisson distribution from the problem above. Indicate its numerical mean and standard deviation in their places on the sketch. Label the horizontal axis "x = food complaints."

Approx normal with mean 259.2 and s.d. = \sqrt{259.2}.

The approximation is better for large \( \mu \) but is rather good for even a mean as small as 3. The s.d. of a Poisson distribution is in general equal to the square root of its mean.

The final 3 questions refer OIL example and E X = expected net return. Use your SLD to select specifications below.

<table>
<thead>
<tr>
<th></th>
<th>for SLD 0, 1, 2</th>
<th>for SLD 3, 4, 5</th>
<th>for SLD 6, 7, 8, 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(OIL)</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>P(+ OIL)</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>P(+ \overline{OIL})</td>
<td>0.3</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>cost to test</td>
<td>20</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>cost to drill</td>
<td>70</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>get from oil</td>
<td>500</td>
<td>500</td>
<td>800</td>
</tr>
</tbody>
</table>

[2 Pts] Fill out a complete tree diagram with all branches and endpoints properly labeled and their numerical values calculated and placed on the tree.

OIL 0.1
- \text{OIL} 0.4 \quad \text{OIL and - 0.04}
+ \text{no OIL} 0.3 \quad \text{no OIL and + 0.27}

\text{OIL and + 0.06}
- 1 no OIL 0.7 no OIL and - 0.63 total = 1.0

Note: The probabilities 0.06, 0.04, 0.27, 0.63 could be used to give the Venn diagram.

[2 Pts] Let r.v. X denote net return from the policy which tests but only drills if the test is positive. Fill out values x (net returns) for all endpoints in your tree diagram above.

Reading from the top these are

-20 -70 +500 = 410
-20 - 0 + 0 = -20
-20 -70 + 0 = -90
-20 - 0 + 0 = -20

[2 Pt.] Calculate E(X)=expected net return. Show your work and reduce your answer to a number.

E NET = 410 (.06) - 20 (.04) - 90 (.27) - 20 (.63) = - 13.1.