**EXAMS 1 THROUGH 3**

Each of the exams covers different material and to a large degree involves differing skills. The information below tracks the performance of 20 students. They are not a random sample but instead belong to one section of 315. Scores are listed in the order of a fixed enumeration of the 20 students. so the first entry of x1 is the raw score of student one on exam 1 and the third score of y2 is the after-scaling score of student three on exam 2.

**DATA BEFORE SCALING (raw scores)**

x1 = exam one raw scores
x2 = exam 2 raw scores
x3 = exam 3 raw scores

\[
\begin{align*}
\text{In}[31] & := x1 = \{15.5, 10, 13, 12, 12, 18, 11.5, 16.5, \\
& \quad 8.5, 13.5, 12, 13.5, 11.5, 20, 9, 15.5, 17, 9.5, 13, 19\} \\
\text{Out}[31] & := \{15.5, 10, 13, 12, 12, 18, 11.5, 16.5, 8.5, \\
& \quad 13.5, 12, 13.5, 11.5, 20, 9, 15.5, 17, 9.5, 13, 19\} \\
\text{In}[32] & := x2 = \{18, 17, 17.5, 12, 18.5, 13, 16.5, \\
& \quad 20, 9, 18, 10, 16, 13.5, 19.5, 14, 19, 17.5, 14.5, 18, 20\} \\
\text{Out}[32] & := \{18, 17, 17.5, 12, 18.5, 13, 16.5, 20, 9, \\
& \quad 18, 10, 16, 13.5, 19.5, 14, 19, 17.5, 14.5, 18, 20\} \\
\text{In}[33] & := x3 = \{13, 5.5, 16, 7.5, 18, 13.5, 17.5, 15, \\
& \quad 6, 16, 6.5, 16, 10, 21.5, 10.5, 10.5, 14, 9.5, 21.5, 17.5\} \\
\text{Out}[33] & := \{13, 5.5, 16, 7.5, 18, 13.5, 17.5, 15, 6, \\
& \quad 16, 6.5, 16, 10, 21.5, 10.5, 10.5, 14, 9.5, 21.5, 17.5\} \\
\text{In}[4] & := \text{mean}[x_] := \text{Apply}[\text{Plus}, x] / \text{Length}[x] \\
\text{In}[5] & := \text{var}[x_] := \text{mean}[(x - \text{mean}[x])^2] \\
\text{In}[45] & := \text{sig}[x_] := \text{Sqrt}[\text{var}[x]] \\
\text{In}[34] & := \{\text{mean}[x1], \text{var}[x1]\} \\
\text{Out}[34] & := \{13.525, 10.5369\} \\
\text{In}[7] & := \{\text{mean}[x2], \text{var}[x2]\} \\
\text{Out}[7] & := \{16.075, 9.98188\} \\
\text{In}[8] & := \{\text{mean}[x3], \text{var}[x3]\} \\
\text{Out}[8] & := \{13.275, 22.6119\}
\end{align*}
\]

The above shows that Exam 3 scores vary more than twice as much as the scores of other exams. The material of chapters 4 and 5 is harder for some students to learn than is the material of chapters 2 and 3. For many it takes some time.

Attendance was much worse during the preparation period for exam 3 than it was for the other exams. In Lecture 2, for example, barely a third of the class...
attended the class Wednesday before break, as might be expected. But barely a third attended the class on the first Monday following break. This, in spite of a good showing on exam 2. After spring break attendance never recovered.

\[ \text{In[9]} := \text{var[\{mean[x1], mean[x2], mean[x3]\}]} \]

\[ \text{Out[9]} = 1.60056 \]

\[ \text{In[10]} := \text{mean[\{var[x1], var[x2], var[x3]\}]} \]

\[ \text{Out[10]} = 14.3769 \]

Decomposition of variance says that the variance of all 60 scores (pooled data for all 20 students and all 3 exams) is equal to the sum of variance of the list of three exam averages = 1.60056 PLUS [mean of the three exam variances] = 14.3769. This shows that exam scores vary much more within exams than between exams. We check our totals below.

\[ \text{In[11]} := \text{var[\{mean[x1], mean[x2], mean[x3]\}]} + \text{mean[\{var[x1], var[x2], var[x3]\}]} \]

\[ \text{Out[11]} = 15.9774 \]

\[ \text{In[36]} := \text{var[\{15.5, 10, 13, 12, 12, 18, 11.5, 16.5, 8.5, 13.5, 12, 13.5, 11.5, 20, 9, 15.5, 17, 9.5, 13, 19, 18, 20, 17.5, 12, 18.5, 13, 16.5, 20, 9, 18, 10, 16, 13.5, 19.5, 14, 19, 17.5, 14.5, 18, 17, 13, 5.5, 16, 7.5, 18, 13.5, 17.5, 15, 6, 16, 6.5, 16, 10, 21.5, 10.5, 10.5, 14, 9.5, 21.5, 17.5\}] \]

\[ \text{Out[36]} = 15.9774 \]

**DATA AFTER SCALING**

\[ \text{In[13]} := \text{y1} = (25/18) \times (x1 + 3); \]

\[ \text{In[14]} := 1.0 \text{y1} \]


\[ \text{In[15]} := \text{y2} = (25/18) \times (x2); \]

\[ \text{In[16]} := 1.0 \text{y2} \]


\[ \text{In[17]} := \text{y3} = (25/20) \times (x3 + 3); \]

\[ \text{In[18]} := 1.0 \text{y3} \]


\[ \text{In[19]} := \text{mean[y1], var[y1]} \]

\[ \text{Out[19]} = \{22.9514, 20.3258\} \]
Variation by exam.

Between exam component of variance.

\( \text{In[22]} := \text{var}\{\text{mean}[y1], \text{mean}[y2], \text{mean}[y3]\} \)

\( \text{Out[22]} = 1.2357 \)

Within exam component of variance.

\( \text{In[23]} := \text{mean}\{\text{var}[y1], \text{var}[y2], \text{var}[y3]\} \)

\( \text{Out[23]} = 24.9707 \)

Variation by student.

Between student component of variance.

\( \text{In[24]} := \text{var}\{\text{Table}[\text{mean}[y1[[i]], y2[[i]], y3[[i]]], \{i, 20\}]\} \)

\( \text{Out[24]} = 18.2932 \)

Within student component of variance.

\( \text{In[25]} := \text{mean}\{\text{Table}[\text{var}[y1[[i]], y2[[i]], y3[[i]]], \{i, 20\}]\} \)

\( \text{Out[25]} = 7.91318 \)

Technical programming.

\( \text{In[27]} := \text{a} = \text{Table}[i, \{i, 20\}] \)

\( \text{Out[27]} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \)

\( \text{In[28]} := \text{Do}[\text{a}[[i]] = \{x1[[i]], x2[[i]], x3[[i]]\}, \{i, 20\}] \)

\( \text{In[29]} := \text{Do}[\text{a}[[i]] = \{y1[[i]], y2[[i]], y3[[i]]\}, \{i, 20\}] \)