

EXAM 1

STT 315 Sp 06

Section number

TA name

Student Number

Print name

Sign name

- Sit where you are asked to.
- Wait for the signal to begin.
- The exam lasts 45 min.
- Remain seated until given permission to move about. No exceptions.
- No extra papers, no calculators, cell phones put away.
- You must neither take the exam apart nor write on anything else.
- Stop writing at once when the signal is given and pass exam ahead.
- Keep your eyes on you own work. No talking.
- Avoid the appearance of flagrantly leaving your paper open to view.
- Point penalties will be exacted for answers given without substantiation.
- Point penalties will be exacted for writing after the signal to stop.
- Any person arriving more than 5 min late will not take the exam.
- Any person leaving without permission will be failed for the course.
- Any person present in a section exam but not enrolled there will be failed.
- Leave fractions unevaluated and do not reduce them.
- Points will be withdrawn for sloppy work.
- Show work in spaces provided. Record your answers in boxes provided.

$$\mathbf{P(OIL) = 0.3, P(+ | OIL) = 0.7, P(+ | OIL^c) = 0.2.}$$

1. Determine $P(-)$.

$$P(OIL -) + P(\text{noOIL} -)$$

$$P(OIL) P(- | OIL) + P(\text{noOIL}) P(- | \text{noOIL})$$

$$= .3 \cdot .3 + .7 \cdot .8 \quad (\text{e.g. } P(- | \text{noOIL}) = 1 - .2)$$

2. Determine $P(OIL | -)$.

$$P(OIL -) / P(-)$$

$$= .3 \cdot .3 / (.3 \cdot .3 + .7 \cdot .8)$$

$$\mathbf{P(OIL) = 0.5, P(OIL +) = 0.20.}$$

3. Determine $P(OIL-)$.

$$P(OIL) = P(OIL+) + P(OIL-)$$

$$0.5 = 0.2 + P(OIL-)$$

$$\text{so } P(OIL-) = 0.3$$

4. Determine $P(- | OIL)$.

$$P(OIL-) / P(OIL)$$

$$= 0.3 / 0.5$$

Balls will be selected without replacement from $\{B B B Y Y\}$

5. Determine $P(B_2)$ using total probability (show all steps taking into account draw 1).

$$P(B_1 B_2) + P(Y_1 B_2)$$

$$= \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4}$$

6. Determine $P(Y_1 B_2 B_3)$.

$$P(Y_1) P(B_2 | Y_1) P(B_3 | Y_1 B_2)$$

$$= \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$$

$P(\text{rain Sat}) = 0.7, P(\text{rain Sun}) = 0.9$, these two events are independent

7. Determine $P(\text{rain Sat AND rain Sun})$.

$$P(\text{Sat}) P(\text{Sun} | \text{Sat}) = P(\text{Sat}) P(\text{Sun}) \quad (\text{by independence})$$

$$= 0.7 \cdot 0.9$$

8. Determine $P(\text{rain Sat OR rain Sun})$.

$$P(\text{Sat}) + P(\text{Sun}) - P(\text{Sat Sun})$$

$$= 0.7 + 0.9 - 0.7 \cdot 0.9$$

Account imbalance X is normally distributed with expectation 100 and sd 2.

9. Determine the standard score of $x = 102.22$ (by hand).
 $(102.22 - 100) / 2 = 2.22 / 2 = 1.11$
10. Determine $P(100 < X < 102.22)$ using the Z method (no continuity correction).
 $z \quad .01$
 1.1 0.3665

r.v. X with $p(0) = 0.25$, $p(2) = 0.5$, $p(4) = 0.25$.

11. Determine $E X^2$
 $E X^2 = \sum x^2 p(x) = 0^2 0.25 + 2^2 0.5 + 4^2 0.25 = 6$
12. Determine $\text{Var } X$.
 $E X = \sum x p(x) = 0 \cdot 0.25 + 2 \cdot 0.5 + 4 \cdot 0.25 = 2$
 $\text{Var } X = E X^2 - (E X)^2 = 6 - 4 = 2$

r.v. X_1, \dots, X_{100} are independent samples of accounts with $E X = 5$, $\text{Var } X = 9$.

13. Determine $E (X_1 + X_2 + \dots + X_{100})$.
 $100 E X = 500$ (on the average, the total of 100 plays is 500)
14. Determine $\text{sd} (X_1 + X_2 + \dots + X_{100})$ (first get the variance).
 $\text{Var}(\text{total of 100 indep plays}) = 100 \text{Var } X = 900$
 $\text{sd}(\text{total of 100 independent plays}) = 30$ (root of variance)

data {3, 4, 5}

15. Determine the sample sd s for the above data.
 mean is 4
 root of $\frac{1}{3-1} ((3-4)^2 + (4-4)^2 + (5-4)^2)$
 $= \sqrt{1} = 1$
16. Determine the sample mean \pm margin of error.
 $4 \pm 1.96 s / \sqrt{n}$
 $4 \pm 1.96 \cdot 1 / \sqrt{3}$

The expected number of raisins in a cookie is 4 and the mix is random.

17. Sketch the normal approximation of the distribution of the number of raisins in a cookie (w/ labels).
 mean 4, $\text{sd} = \sqrt{\text{mean}} = 2$ (for Poisson) (draw normal curve)
18. Determine $p(2)$, the probability that a cookie contains exactly two raisins.
 $p(2) = e^{-\text{mean}} \text{mean}^2 / 2! = e^{-4} 4^2 / 2! = 8 e^{-4}$

A with – repl sample of 400 voters will be selected from a population of which 20 % favor a particular ballot proposal. Let *r.v.* X denote the number of voters in the sample favoring this proposal.

19. Determine the mean and s.d. of X .

Binomial $n = 400$, $p = 0.2$ is the distribution of X .

mean = $np = 80$

var = $npq = 400 \cdot 0.2 \cdot 0.8 = 64$

sd = $\sqrt{\text{var}} = 8$

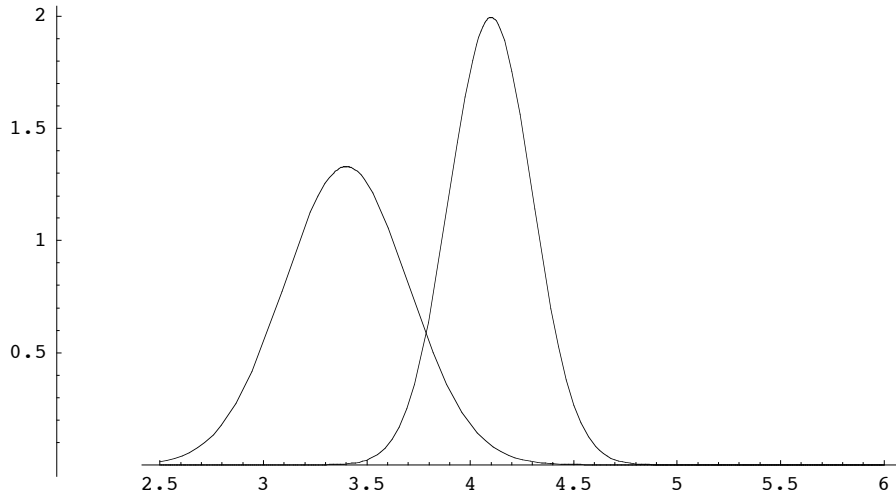
Wait times for customers follows an exponential distribution with mean 5 min.

20. Determine the probability that a customer waits longer than 10 minutes. You need not compute it.

$$e^{-x/\text{mean}} = e^{-10/5} = e^{-2}$$

data {3.4, 4.1}

21. Determine the density portrait for the above data using the figure below.



ans. Obtain the average height of the two curves (midway between them) at a few points on the horizontal axis then join these with a smooth curve.