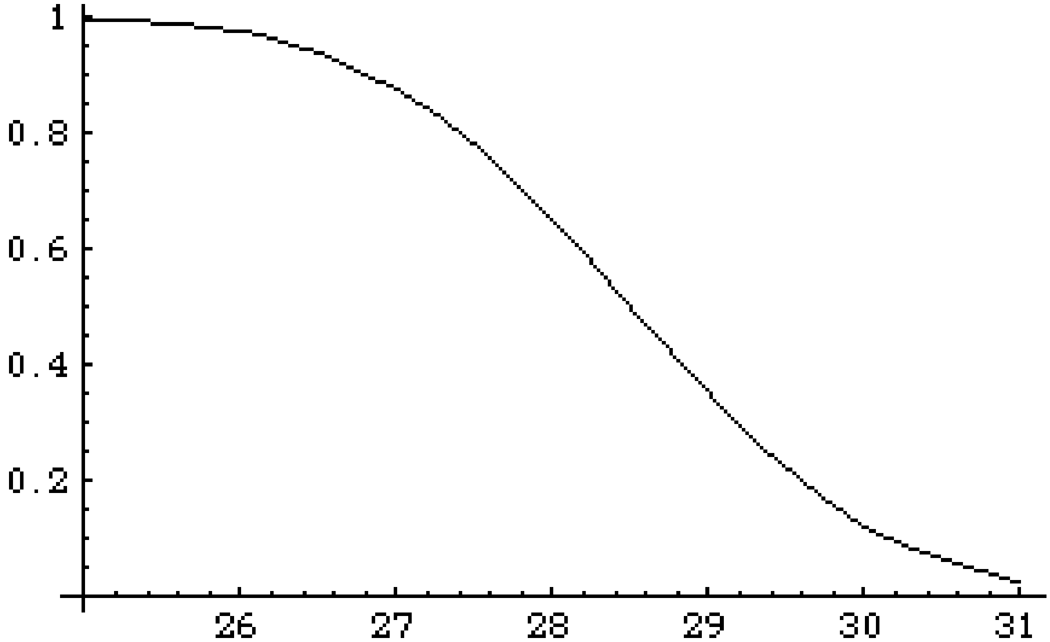
1. Here is a plot of P(reject H0 | mu) as mu varies between 25 and 31. The boundary between H0 and H1 is mu0 = 30.




H1 is where P(reject H0 | mu) is large ->|<- H0

a. 1pt Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the plot.

**ANS. Height of curve at boundary = alpha ~ 0.12 (just be close).**

b. 1pt Give a fairly accurate numerical value for the power at mu1 = 28. Illustrate what you are doing in the curve.

**ANS. Height of curve at mu1 = 28 (which is in H1) = power ~ 0.66 (just be close).**

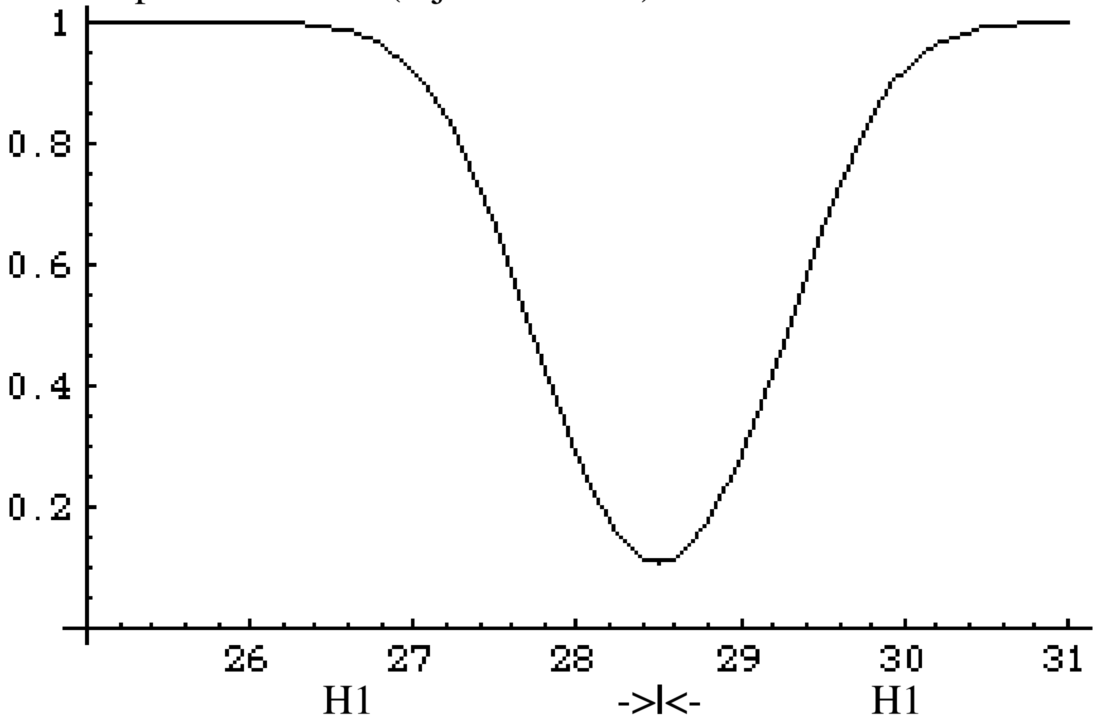
c. 1pt Identify the null and alternative hypotheses.

**ANS. H0 = {mu at least 30} (where the curve is low).**
**H1 = {mu < 30} (the boundary is always put in H0).**

d. 1pt In the above plot, overlay another curve representing P(reject H0 | mu) for a BETTER test of these hypotheses with the SAME ALPHA.

**ANS. Higher everywhere on H1 and lower everywhere on H0 except at the boundary 30 where it is the same height. It is then suggestive of a better curve for the same alpha.**

2. The plot below is P(reject H0 | mu) for mu between 25 and 31.




a. 1pt Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the curve.

**ANS. Height of curve at boundary 28.5 = alpha ~ 0.1 (just be close). Draw lines for this in the picture!**

b. 1pt Give a fairly accurate numerical value for BETA (not power) at mu1 = 27.6. Illustrate what you are doing.

**ANS. Height of curve at boundary = power ~ 0.6 (just be close). So BETA, the probability of failing to reject H0 when mu is 27.6 = 1 – 0.6 = 0.4 (just be close). Draw lines for this in the picture!**

c. 1pt Identify the null and alternative hypotheses.

**ANS. H0 = {mu = 28.5} (where the curve is low).**

**H1 = {mu is not equal to 28.5} (where the curve is high).**

d. 1pt Overlay on the above plot a curve representing a BETTER test for the SAME ALPHA.

**ANS. The better curve is everywhere larger than the one given except that on mu = 28.5 it is 0.1 (alpha is the same).**

3. 2 pts Calculate sample standard deviation "s" for the data {4, 5, 12}. Leave your calculation unevaluated. There is **no partial credit** for this question.

**ANS. xBAR = (4+5+12)/3 = 7**

**s = root( [ $(4\text{-}7)^2$ + $(5\text{-}7)^2$ + $(12\text{-}7)^2$ ] / [3-1] ) = root(19)**

4. a. 1pt A TWO-sided z-test with alpha = 0.05 rejects H0 if the absolute value of the test statistic exceeds which (tabled) value?

**ANS. z(alpha / 2) = z(0.025) = 1.96.**

b. 1pt Consider a ONE sided t-test of H0: mu less or equal 16 ounces, with n = 6 and alpha = 0.01. Evaluate the threshold for rejection using this test (note if it is positive or negative).

**ANS. threshold = + t(alpha) for DF 5 = 3.365. Since H0 is to the left we reject it for large values of xBAR, hence the +t.**

c. 1pt Refer to (4b). If the test statistic turns out to equal 2.9 what action is taken by the test?

**ANS. Fail to reject H0 since the test statistic does not exceed 3.365.**

d. 1pt Refer to (4b). The initial sample of n0 = 6 has sample standard deviation s0 = 4.8. What is the recommended total sample size n to achieve alpha = 0.01 and beta = 0.05 at mu = 16.2? Do not evaluate. Remember, the test is one-sided.

**ANS. t0 = t(alpha) = t(0.01) = 3.365 in the one-sided case.**
**t1 = t(beta) (always, even for two sided) = t(0.05) = 2.015.**
**n = $\left( (3.365 + 2.015)\ 4.8 / (16 - 16.2) \right)^2$**

e. 1pt Refer to (4d). Suppose the required sample size is n = 1345 (it is not). If the sample mean of all 1345 is xBAR = 16.7 what is the value of the HYBRID test statistic; when reject?

**ANS. test statistic = (16.7-16) / (4.8 / root(1345) ) which uses the overall sample mean 16.7 of all 1345, and root(1345), but retains the initial s0 = 4.8 and threshold 3.365. It will reject H0 if the test statistic exceeds 3.365.**

5. a. 2pts Sketch the two-point population distribution for a population having 80% males (each scored x = 1, females scored x = 0) with its sd (give the numerical value). Indicate the population sd by a horizontal line segment laid out to one side of the mean p = 0.8. Overlay on your sketch a density of the sample mean pHAT for n = 400, taking account of the specific population sd which you indicated. This question has no partial credit.

**ANS. Sketch has population distribution portrayed as a stick of height 0.8 above score x = 1 (men) and a stick of height 0.2 above score x = 0 (women). The mean is p = 0.8 with sd = root(.8 .2). Overlay a bell curve of mean = p = 0.8 with sd root(p q) / root(400) = 0.4 / 20 = 0.02. It is the approximate distribution of the sample proportion of males pHAT.**

b. 1pt Imagine that you have a sample SAM of n = 400 with replacement of which 300 (75%) are males. Overlay on your sketch (5a) the LIKELY RESULT of making a density portrait of a list of 10000 sample fractions pHAT*, each such sample fraction being obtained by independently sampling **400** from SAM with-replacement. Be sure to label your curves!

**ANS. If pHAT* values are found from many thousands of samples of 400 from SAM (each with replacement) then most likely these thousands of pHAT* have a density portrait that is roughly normal with mean 300 / 400 = 0.75 and having roughly the sd of normal curve you plotted above. Most every sample SAM can thus reveal the approximate bell dist of pHAT but locate it around pHAT instead of p.**

c. 1pt If the command ci[Mean, SAM, 0.95] will produce a 95% bootstrap CI for Mean[POP1], what command will produce a 95% bootstrap CI for sd[POP1]?

**ANS. If sd[POP1] computes the sd then ci[sd, SAM, 0.95] will give a 95% bootstrap CI for sd[POP1].**

d. 1pt Refer to (5b). Describe how the 95% bootstrap CI for Mean[POP1] appears in the density (5b).

**ANS. It is the central 95% interval of the latter curve, capturing around 95% of those many thousands of pHAT*.**

6. Typically, 50% of e-customers shop after 6 p.m. EST. We decide to test the null hypothesis H0: p = 0.5 versus H1: p is not 0.5, with alpha = 0.10.

**Note: this is a two sided test with boundary p0 = 0.5.**

a. 1pt A current sample of 100 e-customers finds 46 who shop after 6 p.m. Determine the numerical value of the test statistic based upon pHAT (which you must identify). Reduce the test statistic to a number and be sure to say if it is positive or negative.

**ANS. (pHAT-0.5) / (root(p0 q0) / root(n))**
**= (0.46-0.5)/(root(0.5 0.5) / root(100)) = - 0.8 (negative)**

**Note: For 0-1 data we use p0, not pHAT, in the denominator.**

b. 1pt Determine the rejection threshold of a z-test for (6a) and state which action, reject H0 or fail to reject H0, is taken for this data.

**ANS. For a two sided z-test the threshold is z(alpha/2) = z(0.05) = 1.645. Reject H0 if the ABSOLUTE VALUE of the test statistic exceeds 1.645. Since 0.8 does not exceed 1.645 the test fails to reject H0.**

7. A test statistic for a **z-test** evaluates (from the data) to -2.27.

a. 1pt If the hypothesis is H0: p (= or) > 0.3 and the alternative is H1: p < 0.3 what is the numerical value of pSIG?

**ANS. For this H0 we reject for small values of the test statistic. Even more disagreement with H0 would be offered for test statistic < -2.27.**

**So pSIG = P(Z < -2.27) = P(Z > 2.27) = 0.5 - 0.4884.**

b. 1pt Refer to (7a). If pSIG = 0.12 and alpha = 0.01 what action is taken by the z-test and why?

**ANS. When pSIG and alpha are given we reject H0 if pSIG < alpha. That is, reject when an unusually rare disagreement with H0 has been seen in the data. Since pSIG = 0.12 is not less than alpha = .01 we fail to reject H0. Our test, having such small alpha, requires more convincing evidence to abandon H0.**