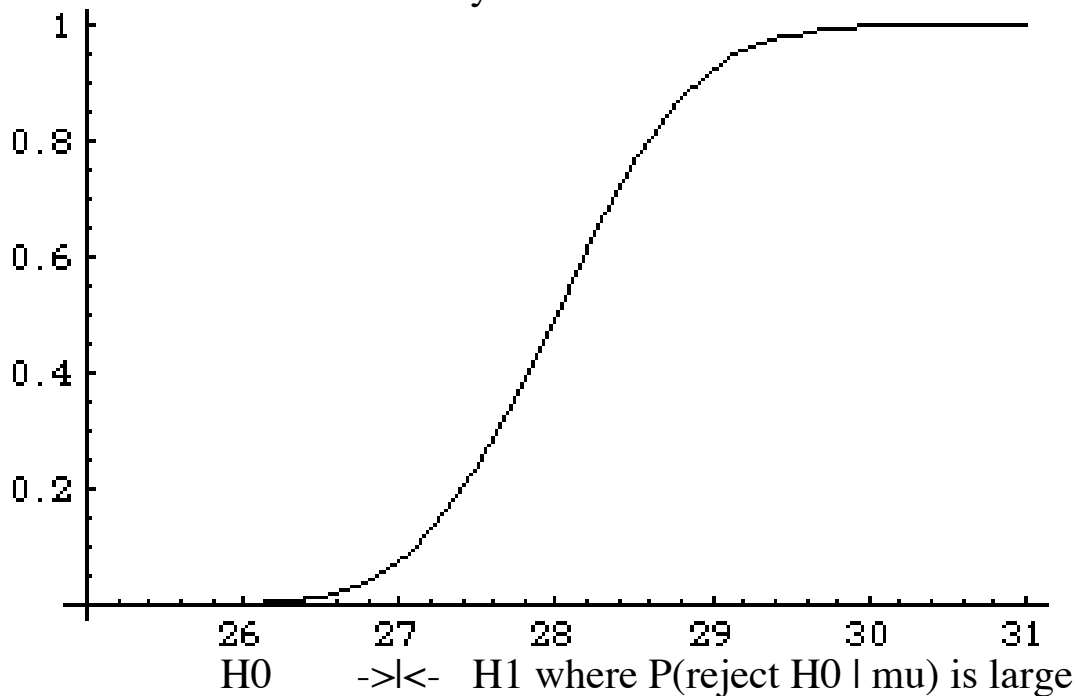1. Here is a plot of P(reject H0 | mu) as mu varies between 25 and 31. The boundary between H0 and H1 is mu0 = 27.

26   27   28   29   30   31
H0   ->|<- H1 where P(reject H0 | mu) is large

a. 1 pt Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the curve.

ANS. Height above the boundary value mu$_0$ = 27 is alpha ~ 0.15 (just be close).

b. 1 pt Give a fairly accurate numerical value for the power and beta at mu1 = 28. Illustrate what you are doing in the curve.

**ANS. power = height at mu1 = 28 is around 0.47. So beta ~ 0.53 (just be close).**

c. 1pt Identify the null and alternative hypotheses.

ANS.   H0 = {mu less or equal 27} where reject probability is low; H1 = {mu > 27} where reject probability is high.

d. 1pt In the above plot, overlay another curve representing P(reject H0 | mu) for a BETTER test of these hypotheses with the SAME ALPHA.

ANS. Overlay a curve that is higher everywhere on H1 and lower everywhere on H0 except at mu$_0$ where it is the same alpha = 0.1 (illustrating having better performance at the same alpha).

2. The plot below is P(reject H0 | mu) as mu varies between 25 and 31.
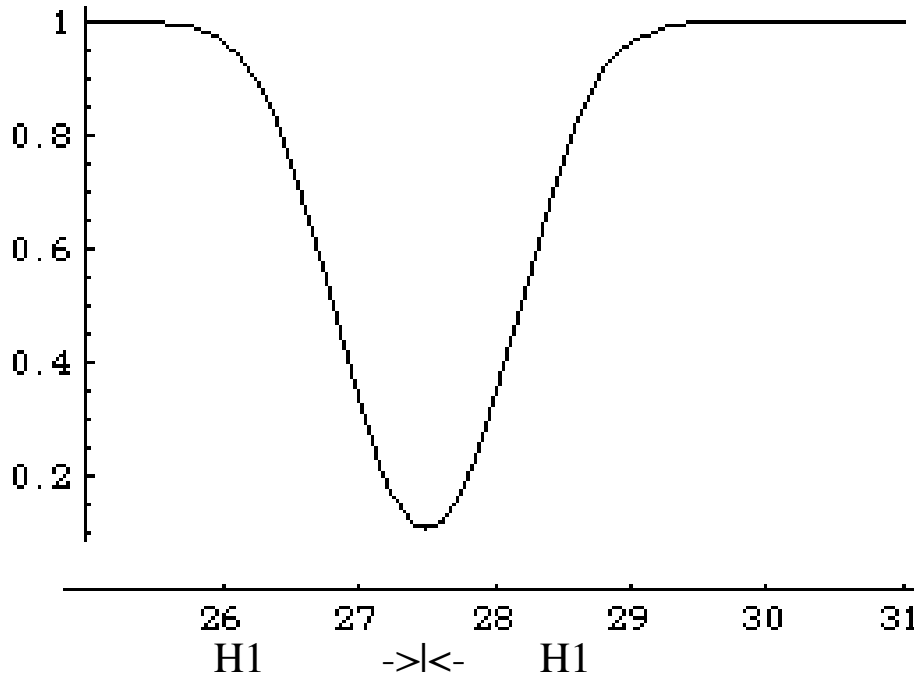
a. 1pt Give a fairly accurate numerical value for alpha. Illustrate what you are doing in the curve.

ANS. This is evidently a two sided test and alpha is the height of curve at $mu_0 = 27.5$ which is alpha ~ 0.12 (just be close).

b. 1pt Give a fairly accurate numerical value for power and beta at mu1 = 28.5. Illustrate what you are doing in the curve.

ANS. Power at mu1 = 28.5 is the height there. It is around 0.78 (just be close). Beta = 1 - power ~ 0.22.

c. 1pt Identify the null and alternative hypotheses.

ANS. H0 = {mu = 27.5}.

H1 = {mu is not equal to 27.5}

d. 1pt Overlay on the above plot a curve representing a BETTER test for the SAME value of alpha.

ANS. Overlay a curve everywhere higher than the given one except having the same height at $mu_0 = 27.5$ (preserving alpha).

3. 2pts Calculate sample standard deviation "s" for the data {4, 5, 11, 12}. Leave your calculation unevaluated. There is **no partial credit** for this question.
ANS. The sample mean is xBAR = (4+5+11+12)/4 = 8.
$$s = root([(4\text{-}8)^2 + (5\text{-}8)^2 + (11\text{-}8)^2 + (12\text{-}8)^2] / [4\text{-}1])$$
$$= root(50 / 3).$$

4.   a. 1pt A (particular) ONE-sided z-test with alpha = 0.05 rejects H0 if the test statistic EXCEEDS which (tabled) value?
ANS. z(alpha) = z(0.05) = 1.645.
     b. 1pt Consider a TWO-sided t-test of H0:   mu = 16 ounces, with n = 10 and alpha = 0.1. Evaluate the **t**-threshold for rejection.
ANS. DF = 9 and t(alpha / 2) = t(0.05) = 1.833.
     c. 1pt Refer to (4b). If the test statistic turns out to equal 1.9 what action is taken by the test?
ANS. Two sided test rejects H0 if ABSOLUTE VALUE of test statistic exceeds threshold 1.833. So it does reject H0.
     d. 1pt Refer to (4b). The initial sample of n0 = 10 has sample standard deviation s0 = 6.1. What is the recommended total sample size n to achieve alpha = 0.1 and beta = 0.025 at mu = 16.5? Do not evaluate. Remember, the test is TWO-sided.
ANS. For the two sided test t0 = t(alpha / 2) = 1.833 but t1 = t(beta) = t(0.025) = 2.262, even for the two sided test. Then
$$n = ((|t0| + |t1|) \, s0 / (mu0 - mu1))^2$$
$$= ((1.833 + 2.262) \, 6.1 / (16 - 16.5))^2$$
     e. 1pt Refer to (4d). Suppose the required sample size is n = 1535 (it is not). If the sample mean of all 1535 is xBAR = 17.9 what is the value of the HYBRID test statistic (do not evaluate)? Under which condition will H0 be rejected?
ANS. T hybrid test statistic (17.9 − 16) / (6.1 / root(1535)) uses xBAR from all n = 1535, and root(1535), but sticks with s0 = 6.1 from initial sample of 10. This two sided test rejects H0 if the ABSOLUTE VALUE of the test statistic exceeds 1.833 (determined, recall, from the initial sample size 10).

5.   a. 2pts Sketch a hypothetical population density for a population POP1 having mean 56, with its sd 22 indicated by a horizontal line segment laid out to one side of the mean 56 (this need not be exact by any means, since general distribution shapes do not have readily recognizable sd). Overlay on your sketch a normal density (suggested by the CLT as an approximation) for the sample mean xBAR for n = 25.

ANS. The population could be sketched as perhaps having some few modes, to emphasize a possible lack of normality. Locate the mean 56 at the balance point and make a stab at representing the sd 22, although there is no easy way to visualize the sd reliably. Around this mean 56 place a normal density having sd = 22 / root(25) = 4.4. This is the CLT approximation of the distribution of xBAR for n = 25.

      b. 1pt Imagine that you have a sample SAM of n = 25 with replacement and the sample mean of SAM is 56.5. Overlay on your sketch (5a) the LIKELY RESULT of making a density portrait of a list of 10000 sample means xBAR*, each such sample mean being obtained by independently sampling 25 from SAM with-replacement. Label your curves as clearly as you can.

ANS. This is the central idea of bootstrap. Your curve (5a) represents the variations of possible xBAR around mu. The curve you will overlay will resemble it in shape but will instead be centered on xBAR = 56.5 (from your sample of 25). The latter curve portrays the variations of xBAR* around 56.5.

      c. 1pt If the command   ci[Mean, SAM, 0.95] will produce a 95% bootstrap CI for Mean[POP1], what command will produce a 95% bootstrap CI for Median[POP1]?

ANS.   If Median[POP1] calculates the median then the bootstrap 95% CI for Median[POP1] is called by ci[Median, SAM, 0.95].

      d. 1pt Refer to (5b). Describe how the 95% bootstrap CI for Mean[POP1] appears in the density (5b).

ANS. It is the central 95% interval of curve (5b) which captures the central 95% xBAR* values around xBAR.

6. Typically, 50% of e-customers shop after 6 p.m. EST. We decide to test the null hypothesis H0: p (< or =) 0.5 versus H1: p > 0.5, with alpha = 0.05.

   a. 1pt A current sample of 100 e-customers finds 46 who shop after 6 p.m. Determine the numerical value of the test statistic based upon pHAT (which you must identify). Reduce the test statistic to a number and be sure to say if it is positive or negative.
ANS. Test statistic (pHAT – 0.5) / (root(0.5 0.5) / root(100))
= (0.46 – 0.5) / (0.5 / 10) = - .04 (20) = -0.8

   b. 1pt Determine the rejection threshold of a z-test for (6a) and state which action, reject H0 or fail to reject H0, is taken.
ANS. For this one sided test we reject H0 for large values of the test statistic.   Since –0.8 actually favors H0 we have insufficient grounds to reject H0.

7. A test statistic for a **z-test** evaluates (from the data) to 3.14.

   a.   1pt If the hypothesis is H0: p = 0.4, and the alternative is H1: p is not 0.4, what is the numerical value of pSIG?
ANS. For a two sided test pSIG is
    P(|Z| > 3.14) = 2 P(Z > 3.14) = 2 (0.5 – 0.4992)
Always, pSIG is the probability of data more disagreeable with H0 than is our data SAM.

   b.   1pt Refer to (7a). If   pSIG = 0.22 and alpha = 0.32 what action is taken by the z-test and why?
ANS. Always, the rule is to reject H0 if pSIG < alpha. In this case we reject H0.