These notes for the chapter 8 assignment do not provide the solutions. They offer insights into some practical issues raised by the problems.

For **large** positive z the probability zTail[z] = P(Z > z) is rather well approximated by

$$\text{zTail}[z] \sim \frac{1}{z\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}$$

and in fact the ratio of the left and right sides tends to 1 as z tends to infinity. So you have the means to evaluate pSIG for really large z. Some of these exercises have standard scores like 10 or 13. Here e denotes the constant 2.718281828

As shown below, this is not so accurate for z = 1.96, which should give 0.025. But is better for z = 2.576, which (from t-table) should give 0.005. When we move up to z = 5.0, nearly the largest entry of the z-table, we get very close to the 0.0000003 obtained from the table.

```
N[e, 15]
```

```
2.71828182845905
```

$$\left\{\frac{1}{1.96\sqrt{2\pi}}\, e^{-\frac{1.96^2}{2}},\ \frac{1}{2.576\sqrt{2\pi}}\, e^{-\frac{2.576^2}{2}},\ \frac{1}{5.0\sqrt{2\pi}}\, e^{-\frac{5.0^2}{2}}\right\}$$

$\{0.0298168,\ 0.00561079,\ 2.97344\times 10^{-7}\}$

8-2. In the broader sense we are comparing $\overline{D} = 5$ with zero. The appropriate standard score is a whopping 13.749. Whether by test or by CI the evidence is surely very strong.

We shall likely use the z-methods for CI and for testing the hypothesis of no difference between the two makes. This is because 40 is a fairly large sample size and we have no reason to believe the population scores are normal.

In addition to a CI (it seems a little silly to use a 95% CI when the data is so convincing) you should calculate pSIG, which will be tiny indeed, for the appropriate two-sided test.

What is the population? Since the drivers have been selected at random, from some population of drivers, the statistical analysis will apply to that population of drivers.

Could drivers do best with the first car they drive? Perhaps one's senses are sharpest for the first drive. To blunt such criticisms one might arrange it so that each brand is driven first in a randomly selected one half of the 40 tests. Then the population is all possible allocations of first-driven brand for all possible drivers. The score is still d = Mazda time - Nissan time.

If we were using only a fixed set of 40 professional drivers the population would be all possible allocations of first-driven to these fixed drivers.

It is known that the z-test and CI perform well in all three of the applications above.

```
5 / (2.3 / Sqrt[40])
```

```
13.749
```

8-4. The main point is that this z-test is one-sided because we are seeking to show that the program incentives reduce consumption. The data is very convincing. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and conclusion of the test. You should also report pSIG.

```
0.2 / (0.1 / Sqrt[60])
```

```
15.4919
```

8-6. Why are we interested in d = proportion invested HK after Oct. 15 - proportion invested HK before Oct. 15? It seems we are interested in investor's habits regardless of the size of their portfolios. The sample size 25 is perhaps a little small to justify z-methods. As long as you keep $\overline{D} = 4$, $s_d = 2$, and the boundary of the null and alternative hypotheses in the same units you need not worry over how to record these percentage scores. It is not about 0-1 data however, for we are simply recording scores that are expressed as percents. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and conclusion of the test. You should also report pSIG. The evidence seems overwhelming.

```
4 / (2 / Sqrt[25])
```

```
10
```

8-8. The sampling unit is TV program. The score is d = rating by men - rating by women. These scores d are believed to be normally distributed so t-methos are appropriate. You must determine n, $\overline{D}$ and s_d yourself. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and report pSIG. Remember, pSIG represents the chance of getting more evidence against the null hypothesis than this sample has provided, if the true μ_d is equal to the bounbdary of the null and alternative.

8-10. This is a two-sample problem. The target is $\mu_{\text{Nikon}} - \mu_{\text{Minolta}}$, the difference between the mean ratings of the two camera models in the population of all photographers comprising the population from which each sample of 30 was selected. We'll apply the z-method with the caution that the two sample sizes of 30 each are a little lower than we would like. This could be a problem if some few photographers give one or the other camera model unusually low or high scores, such as photographers who cannot abide the lack of any capability to mechanically preview the diaphram closure on the Minolta. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and conclusion of the test. You should also report pSIG. The evidence seems weak.

$$(8.5 - 7.8) \Big/ \text{Sqrt}\left[\frac{2.1^2}{30} + \frac{1.8^2}{30}\right]$$

```
1.38621
```

8-12. I find the statement of the problem vague. We are not clearly told what the nature of the chart is. These mis-givings aside, it is intended to be a two-sample problem. Apply the z-method with the caution that the two sample sizes of 35 each are a little lower than we would like. This could be a problem if some few investment-spikes, up or down, unduly affect the means. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and conclusion of the test. You should also report pSIG. The evidence is not overwhelming although some test at modest α may reject a hypothesis of no difference.

```
(3200 - 2800.) / Sqrt[900^2/35 + 800^2/35]
```

$$(3200 - 2800.) \Big/ \text{Sqrt}\left[\frac{900^2}{35} + \frac{800^2}{35}\right]$$

```
1.96521
```

8-14. You are being invited to apply a two-sample t-test for small sample sizes $n_1 = n_2 = 13$. The two sample means and sd are to be determined from the data given. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and conclusion of the test. You should also report pSIG.

```
hotel = {17, 11, 14, 25, 9, 18, 36, 19, 22, 24, 16, 31, 23}
```

```
{17, 11, 14, 25, 9, 18, 36, 19, 22, 24, 16, 31, 23}
```

```
Length[hotel]
```

```
13
```

```
N[Sqrt[13 / 12] Sqrt[Mean[hotel ^ 2] - Mean[hotel] ^ 2]]
```

```
7.62166
```

8-16. You are being invited to apply a one-sided two-sample z-test. Do state the null and alternative hypotheses and the form and evaluation of the test statistic. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is.

$$(1838.69 - 1050.22) \Big/ \text{Sqrt}\left[\frac{461^2}{100} + \frac{560^2}{80}\right]$$

```
10.141
```

- IN THE FOLLOWING PROBLEMS THE CALCULATIONS HAVE BEEN CORRECTED FROM THE ORIGINAL POSTING IN WHICH A SYSTEMATIC ERROR (AN INAPPROPRIATE SQUARE OF pq VALUES) WAS INDUCED AND PROPAGATED BY CUT AND PASTE. TO BE VERY CLEAR ON THIS POINT, THERE IS A SQUARE JUST ABOVE BECAUSE WE NEED VARIANCES, NOT SDs UNDER THE SQUARE ROOT. IN THE 0-1 DATA CASES BELOW ALL VARIANCES ARE GIVEN IN TERMS OF pq VALUES.

8-30. You are being invited to apply a one-sided two-sample z-test. Do state the null and alternative hypotheses, the form and evaluation of the test statistic and conclusion reached by the test. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is. The evidence is overwhelming.

```
N[{850 / 1000, 1950 / 2500}]

{0.85, 0.78}
```

In[1]:= $(.85 - .78) \Big/ \mathrm{Sqrt}\left[\frac{(0.85\ \ 0.15)}{1000} + \frac{(0.78\ 0.22)}{2500}\right]$

Out[1]= 4.99822

In[6]:= $\frac{1}{4.99822\sqrt{2\pi}}\, e^{-\frac{4.99822^2}{2}}$

Out[6]= 3.00108×10^{-7}

8-32. You are being invited to apply a one-sided two-sample z-test. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and the decision reached. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is. The evidence is insubstantial in comparison with the very large sample sizes. With such values as 13% we are wary of using z for sample sizes like 30 but can be comfortable with such large sample sizes as we have here.

In[2]:= $((.19 - .13) - .05) \Big/ \mathrm{Sqrt}\left[\frac{(0.19\ \ 0.81)}{5000} + \frac{(0.13\ 0.87)}{2060}\right]$

Out[2]= 1.08032

8-34. You are being invited to apply a one-sided two-sample z-test. Do state the null and alternative hypotheses and the form and evaluation of the test statistic. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is.

8-36. You are being invited to apply a two-sided two-sample z-test. I am wary of the values 0.075 and 0.072 which strain the CLT, but for the sample sizes of 1000 are probably ok. Do state the null and alternative hypotheses and the form and evaluation of the test statistic. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is. The evidence is weak.

In[3]:= $((.075) - .072) \Big/ \text{Sqrt}\left[\frac{(0.075\ \ 0.925)}{1000} + \frac{(0.072\ 0.928)}{1000}\right]$

Out[3]= 0.257067

8-38. You are being invited to apply a two-sided two-sample z-test. Do state the null and alternative hypotheses and the form and evaluation of the test statistic and decision reached. You should report pSIG. Remember, what you hope to "prove" is the opposite of the null hypothesis. Here "prove" only means that the opposite of what you promote, the null hypothesis, rarely yields sample data more disagreeable with the null hypothesis than your data is. Taking the orientation East-West, I give below the approximation of P(Z > m) where m is the test statistic. You need it to determine pSIG for this two-sided test. Re-calculate, showing what m is equal to. As you can see, the evidence is far more overwhelming than the $\alpha = 0.05$ test reveals. You might look at a 99% CI for $p_1 - p_2 = \text{East} - \text{West}$ in this case.

In[4]:= $m = ((.551) - .483) \Big/ \text{Sqrt}\left[\frac{(0.551\ \ 0.449)}{1000} + \frac{(0.483\ 0.517)}{1000}\right];$

In[5]:= $$\frac{1}{m\sqrt{2\pi}}\,e^{-\frac{m^2}{2}}$$

Out[5]= 0.00124961