STT 351-001

EXAM 1 KEY

001

1. a. Given independent random variables X, Y with E X = 4, and E Y = 6. Determine the numerical value of E(2X + XY). Do not reduce it.

$$E(2X+XY) = 2EX + E(XY)$$
$$\overset{INDEP}{=} 2(4) + (EX)(EY)$$
$$= 2(4) + 4(6)$$

b. Unrelated to (a). Consider a continuous probability density

f(x) = x/2, 0 < x < 2, zero elsewhere,

Set up, and numerically evaluate, Variance X. Do not reduce it.

$$EX = \int x\, f(x)\,dx = \int_0^2 x\,(x/2)\,dx$$
$$= \frac{x^3}{6}\Big]_0^2 = 8/6 = 4/3$$
$$EX^2 = \int x^2 f(x)\,dx = \int_0^2 x^2\, x/2\, dx$$
$$= \frac{x^4}{8}\Big]_0^2 = \frac{16}{8} = 2$$
$$\mathrm{Var}\, X = EX^2 - (EX)^2$$
$$= 2 - (4/3)^2$$

2. Recall that the Poisson distribution has **variance** equal to its mean.

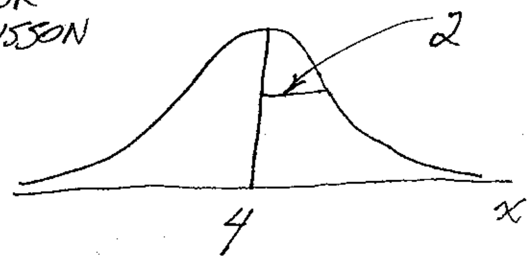
a. Sketch the CLT-approximation of the distribution of random variable X = the number of accidents this month. Assume that X is Poisson distributed and we average around 4 accidents per month. Be sure to label the mean and **s.d.** of X as recognizable numerical elements in your sketch.

$\mu_X = EX = 4$

$\sigma_X = \sqrt{\mathrm{Var}X} = \sqrt{\mu_X} = \sqrt{4} = 2$

FOR ANY r.v.   FOR POISSON

$X \approx$



b. Determine the normal approximation of the probability of **fewer than 7** accidents this month. It is customary to instead approximate P(X < 6.5) (i.e. use the normal approximation with continuity correction). Obtain the relevant z-score and use it to obtain the normal approximation of the probability.

$P(X < 6.5) \approx P\left(Z < \frac{6.5-\mu_X}{\sigma_X}\right) = P\left(Z < \frac{6.5-4}{2}\right)$

$P(0)+\cdots+P(6)$

$P(X<7)$

- DISCRETE -

$= P(Z < 1.25)$

$= 0.8944$

3 $.05$

1.2 $\boxed{0.8944}$

NORMAL APPROX W/ CONTINUITY CORRECTION

NOTE: (NOT REQUIRED ON EXAM)

$\underbrace{P(0)+\cdots+P(6)}_{\text{EXACT } P(X<7) \text{ FOR POISSON W/ MEAN} = 4} = e^{-4}\left(\frac{4^0}{0!}+\frac{4^1}{1!}+\frac{4^2}{2!}+\cdots+\frac{4^6}{6!}\right) = 0.8893$

CLOSE!

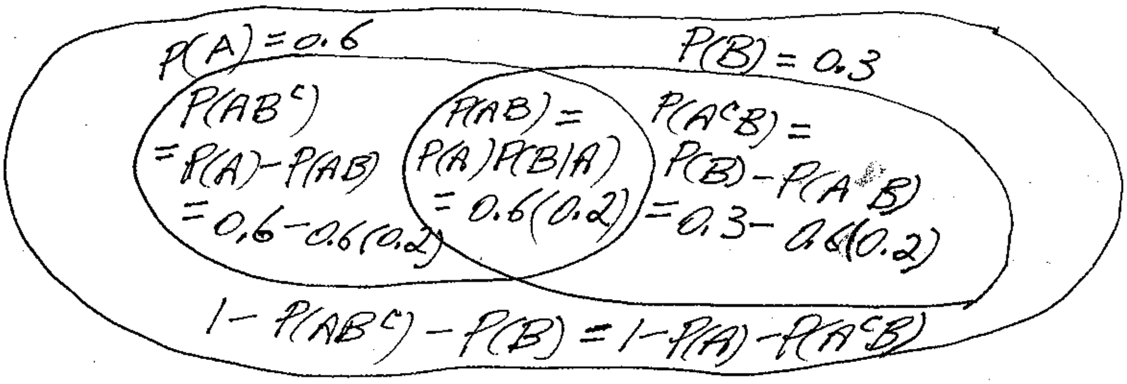
3. Events A, B have P(A) = 0.6, P(B) = 0.3, P(B | A) = 0.2.

a. Determine P(A union B). $= P(A) + P(B) - P(AB)$

∪ "or" ADDN RULE

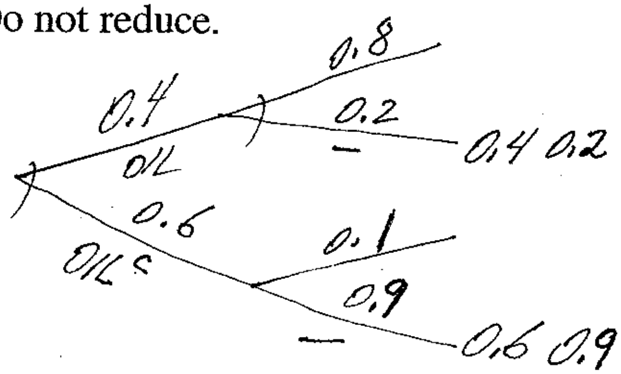
$$= P(A) + P(B) - P(A)P(B|A)$$
$$= 0.6 + 0.3 - 0.6(0.2)$$

b. Complete a Venn diagram with all four regions and their probabilities. Be clear about the basis for your choices.



4. TREE. Initial assessments say the probability of oil at the drill site is 0.4. There is a test for oil. If oil is present there is conditional probability 0.8 the test will show positive. If oil is not present there is conditional probability 0.1 the test will be positive.

a. Determine the probability the test is **negative**.
Do not reduce.



OR simply

$$P(-) = P(OIL\,-) + P(OIL^c\,-)$$
$$= P(OIL)P(-|OIL) + P(OIL^c)P(-|OIL^c)$$
$$= 0.4\ 0.2 + 0.6\ 0.9$$

b. Determine P(oil is present | test is **negative**).
Set up with numbers but do not reduce.

$$P(OIL|-) \underset{DEF}{=} P(OIL\,-)/P(-) = \frac{0.4\ 0.2}{0.4\ 0.2 + 0.6\ 0.9}$$

5. Hours x waiting for a new peak in demand is modeled as a random variable X with P(X > x) = 1/x for each value x > 1.

a. Determine the probability that you will have to wait at least 10 hours for a new peak in demand, i.e. P(X > 10).

$P(X > 10) = 1/10$ i.e $x = 10$.

b. Determine the conditional probability that you will wait at least an **additional** 10 hours for a new peak if you have already waited 20 hours without a new peak, i.e. P(X > 30 | X > 20).

$P(X>30 \text{ and } X>20)/P(X>20)$

$= P(X>30)/P(X>20) = (1/30)/(1/20) = 2/3.$

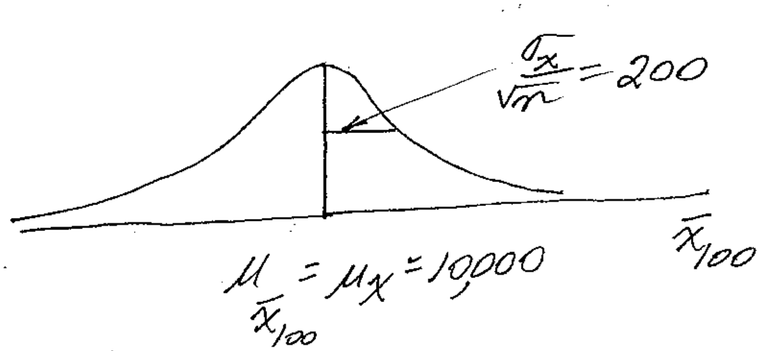
6. A process produces parts scored x = lifetime in lab test conditions. Assume that E X = mu = 10,000 hours with s.d. = sigma = 2,000 hours.

a. Denote by xBAR the sample average of 100 independent sample lifetimes of such parts. Determine numerically, but do not reduce,

E xBAR $= \mu_x = 10,000$

s.d. xBAR $= \frac{\sigma_x}{\sqrt{n}} = \frac{2000}{\sqrt{100}} = 200$

b. Sketch the **approximate** distribution of xBAR as given by the CLT. Indicate the mean and s.d. as recognizable numerical entities in your sketch, but do not reduce them.



7. Balls will be selected without replacement and with equal probability on those then remaining from $\{ \underbrace{R\ R}_{2}\ \underbrace{G\ G\ G\ G}_{4}\ \underbrace{Y}_{1} \}$. TOTAL 7 BALLS

a. Give P(R2 | R1). Compare it with P(R1 | R2), which you must compute using the rules of probability. Solve in no other way.

BY ASSUMPTION $P(R2/R1)$ REFERS TO SELECTION FROM $\{R\ G\ G\ G\ G\ Y\}$, SO $P(R2/R1) = 1/6$

$$P(R1/R2) \overset{DEF}{=} P(R1R2)/P(R2) = \frac{2/7 \cdot \frac{1}{6}}{2/7 \cdot \frac{1}{6} + 5/7 \cdot 2/6}$$

$$P(R1R2) = P(R1)P(R2/R1) = \frac{2}{7}\frac{1}{6}$$

$$P(R2) = P(R1R2) + P(R1^cR2) = 2/7 \cdot \frac{1}{6} + \frac{5}{7}\frac{2}{6}$$

b. Determine P(G**2** or Y**3**). You may use the "order of the deal" principle.

ORDER OF DEAL

$$P(G2 \cup Y3) = P(G1 \cup Y2)$$

$$= P(G1) + P(Y2) - P(G1\,Y2)$$

$$P(Y2) \overset{\text{order of deal}}{=} P(Y1) = \frac{1}{7}$$

$$= 4/7 + \frac{1}{7} - P(G1)P(Y2/G1)$$

$$= 4/7 + 1/7 - \frac{4}{7}\frac{1}{6}$$

~~do~~ not reduce.