2

1. a. Given **independent** random variables X, Y with $E X^2 = 4$, and $E Y = 7$. Determine the numerical value of $E(3 + Y + Y X^2)$. Do not reduce it.

$E(3+Y+YX^2) = 3+EY+EYX^2$

$\overset{\text{INDEP}}{=} 3+EY+EY\,EX^2 = 3+7+7(4)$

b. Unrelated to (a). Consider a continuous probability density

$f(x) = 1/4,\ 0 < x < 4$, zero elsewhere,

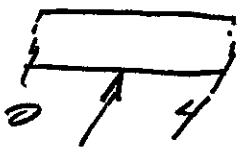
Set up, and numerically evaluate, standard deviation X. Do not reduce it.

KEY 351-002

EXAM 1 MAKEUP

$EX = \int_0^4 \frac{1}{4}\, x\, dx = \frac{x^2}{2\cdot 4}\Big]_0^4$

$= \frac{16}{8} = 2$



$EX^2 = \int_0^4 x^2 f(x)\, dx = \int_0^4 x^2 \frac{1}{4}\, dx = \frac{x^3}{4\cdot 3}\Big]_0^4 = \frac{16}{3}$

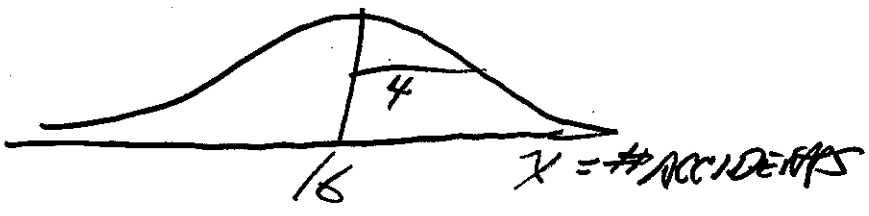
$\sigma_x = \sqrt{\text{Var}\, X} = \sqrt{EX^2 - (EX)^2} = \sqrt{\frac{16}{3} - 2^2}$

2. Recall that the Poisson distribution has **standard deviation** equal to the **square root** of its mean.

a. Sketch the CLT-approximation of the distribution of random variable X = the number of accidents this month. Assume that X is Poisson distributed and we average around 16 accidents per month. Be sure to label the mean and **s.d.** of X as recognizable numerical elements in your sketch.

$\sigma_x \overset{\text{POISSON}}{=} \sqrt{\mu_x} = \sqrt{16} = 4$

$\approx$



b. Determine the normal approximation of the probability of **fewer than 20** accidents this month. It is customary to instead approximate P(X < 19.5) (i.e. use the normal approximation with continuity correction). Obtain the relevant z-score and use it to obtain the normal approximation of the probability.

$P(X<19.5) = P\left(\frac{X-\mu_X}{\sigma_X} < \frac{19.5-\mu_X}{\sigma_X}\right)$

$\approx P\left(Z < \frac{19.5-16}{4}\right) = P\left(Z < \frac{3.5}{4}\right)$

$= P(Z < 0.88)$ ~ 0.87 OR too

$= 0.8106$

or 0.8078

.875

4|3.5

.08

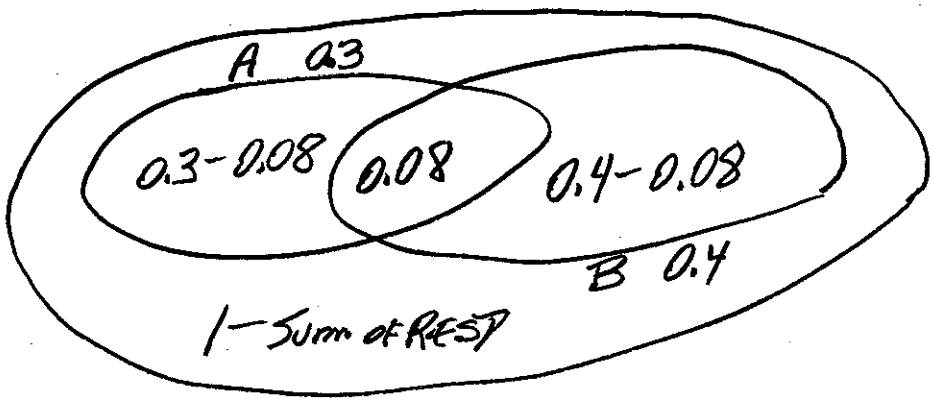
0.8 | 0.8106

3. Events A, B have P(A) = 0.3, P(B) = 0.4, P(A | **B**) = 0.2.

a. Determine P(A intersection B). $= P(AB) = P(B)P(A|B)$

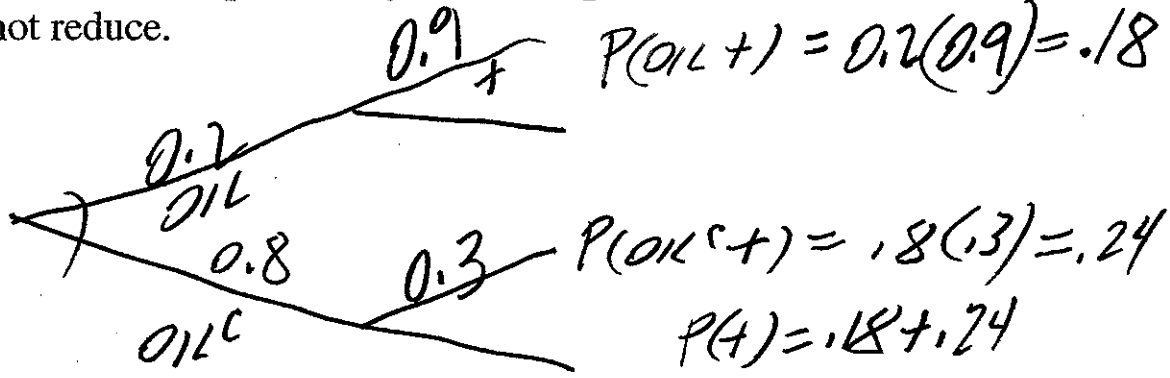
$= 0.4(0.2) = 0.08$

b. Complete a Venn diagram with all four regions and their probabilities. Be clear about the basis for your choices.



4. TREE. Initial assessments say the probability of oil at the drill site is 0.2. There is a test for oil. If oil is present there is conditional probability 0.9 the test will show positive. If oil is not present there is conditional probability 0.3 the test will be positive.

a. Determine the probability the test is **positive**.
Do not reduce.



b. Determine P(oil is present | test is **positive**).
Set up with numbers but do not reduce.

$$P(OIL \mid +) \underset{Def}{=} P(OIL +)/P(+) = \frac{.18}{.18+.24}$$

5. Hours x waiting for a new peak in demand is modeled as a random variable X with $P(X > x) = 1/x$ for each value $x > 1$.

a. Determine the conditional probability that you will wait **at least** an **additional** 8 hours for a new peak if you have already waited 100 hours without a new peak, i.e. $P(X > 108 \mid X > 100)$.

$$P(X>108 \mid X>100) = P(X>108)/P(X>100)$$
$$= (1/108)/(1/100) = \frac{100}{108}$$

b. Determine the probability that you will have to wait **fewer than** 8 hours for a new peak in demand, i.e. P(X < 8).

$P(X<8) = 1 - P(X>8) = 1 - \frac{1}{8} = \frac{7}{8}$

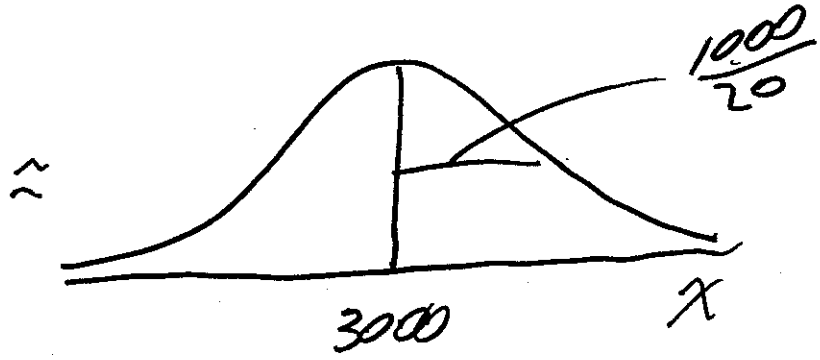
6. A process produces parts scored x = lifetime in lab test conditions. Assume that E X = mu = 3,000 hours with s.d. = sigma = 1,000 hours.

a. Denote by xBAR the sample average of 400 independent sample lifetimes of such parts. Determine numerically, but do not reduce,

E xBAR $= \mu_{\bar{x}} = 3000$

s.d. xBAR $= \frac{\sigma_x}{\sqrt{n}} = \frac{1000}{\sqrt{400}} = \frac{1000}{20}$

b. Sketch the **approximate** distribution of xBAR as given by the CLT. Indicate the mean and s.d. as recognizable numerical entities in your sketch, but do not reduce them.



7. A coin will be flipped. If a head results a ball will be selected from { R R G G G G Y }. If a tail results a ball will instead be selected from { R G G Y }

a. Give P(R | head). $= \frac{2}{7}$ Draw 2 from RR GGGGY

b. Determine P(R).

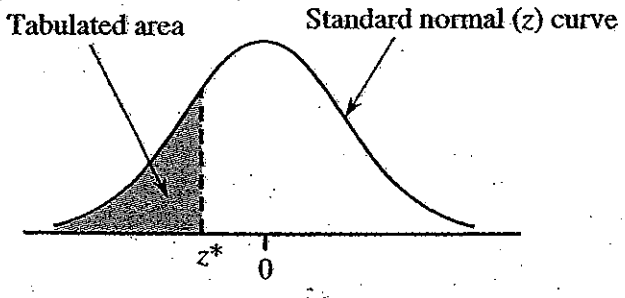
$= P(HR) + P(TR)$

$= P(H)P(R|H) + P(T)P(R|T)$

$= \frac{1}{2}\frac{2}{7} + \frac{1}{2}\frac{1}{4}$

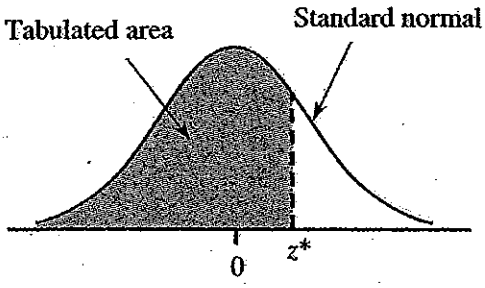
← from RGGY

Table I The standard normal distribution (cumulative z curve areas)



z^*	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.8	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
−3.7	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247

Table I The standard normal distribution *(continued)*



z^*	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999