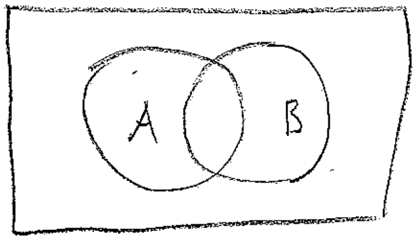
STT351-002 HW #2 [redacted]

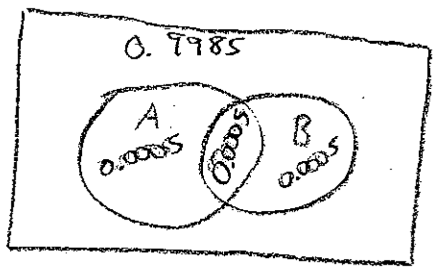
Key

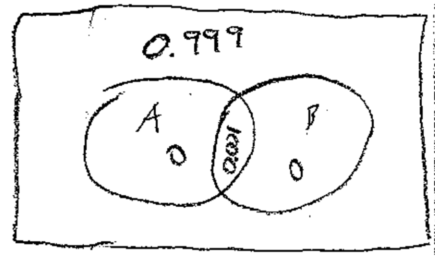
5.17 Two Engines Each requires unscheduled repair with an estimated probability 0.001.



A = first engine requires repair
B = second engine required repair

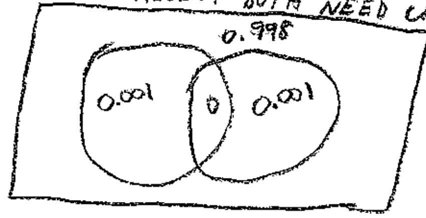
a)





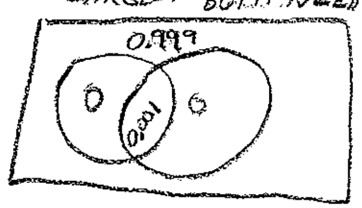
b. $\boxed{0}$

SMALLEST BOTH NEED Unscheduled Repair



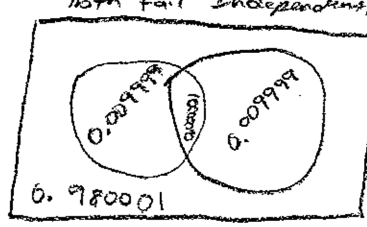
c. $\boxed{0.001}$

LARGEST BOTH NEED Unscheduled Repair



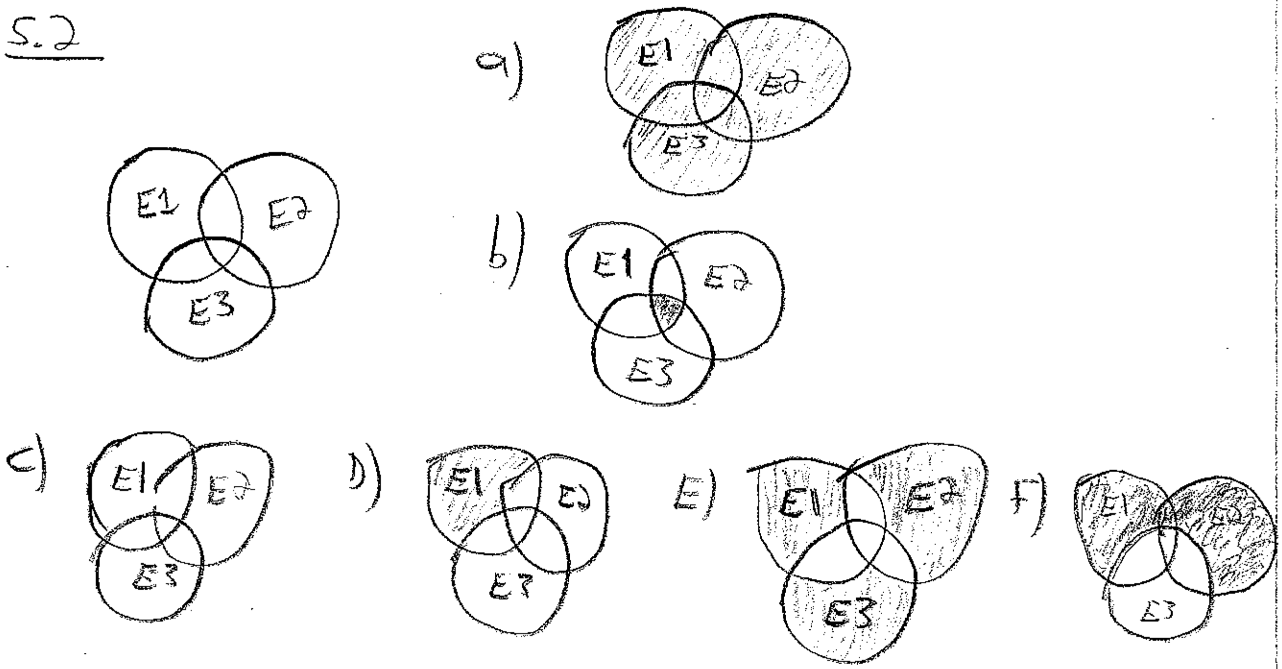
d. $\boxed{0.000001}$

Both fail Independently



STT 351-002 HW#2 [redacted]

5.2



5.8

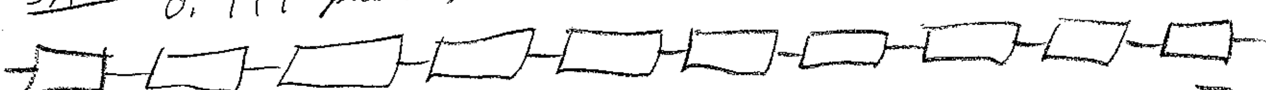
A, B, C, D, E

a) $P(A \text{ or } A) = \frac{1}{5} + \frac{1}{5} = \boxed{\frac{2}{5}} = \boxed{0.4}$

b) $P(A \text{ or } A) = \frac{1}{5} + \frac{1}{5} = \boxed{\frac{2}{5} = 0.4}$

c) ~~For~~ these two types of testing you will always get same probability of finding the defective items, no matter how many samples.

5.10 0.999 probability of functioning without failure



$P(A_1 \text{ and } A_2 \& A_3, \dots A_{10}) \geq 1 - [P(A_1') + P(A_2') + P(A_3') \dots + P(A_{10}')]$

$P(\text{AND}) \geq 1 - [0.001 + 0.001 + 0.001 \dots + 0.001] = 1 - 0.01 = 0.99$

$\boxed{\text{Lower bound} = 0.99}$

STT 351-002 HW #2 [redacted]

5.12

A	B	C	D	E
0.2	0.25	0.15	0.3	0.1

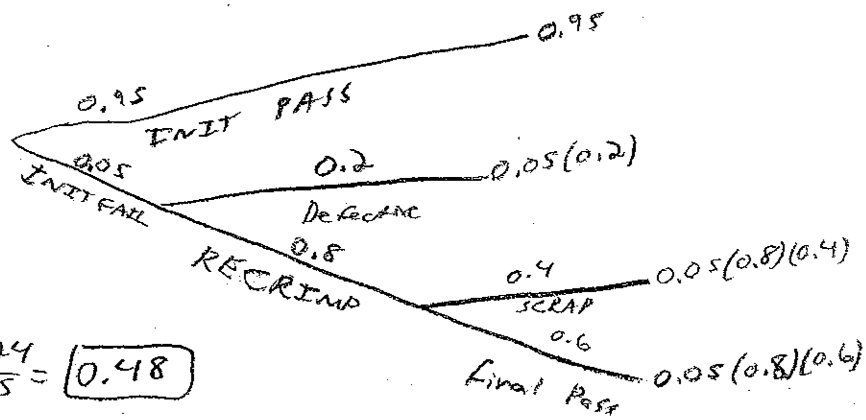
a. ~~If E goes out of business...~~ CONDITIONAL ON E NOT WINNING A BID CONDUCTED WITH PROBABILITIES ABOVE

A	B	C	D
$0.2/0.9 = 0.\overline{222}$	$0.25/0.9 = 0.2778$	$0.15/0.9 = 0.16667$	$0.3/0.9 = 0.\overline{333}$

b. If only A & C Available

A	C
$0.2/0.35 = 0.57143$	$0.15/0.35 = 0.42857$

5.14



a. $\frac{0.024}{0.05} = \boxed{0.48}$

b. $0.95 + 0.024 = \boxed{0.974}$

c. $\frac{0.95}{0.974} = \boxed{0.97536}$

STT 351-002 HW #2 [redacted]

5.18

A	B	AB	O
0.42	0.10	0.04	0.44

a. $p(A|A) = \underline{0.1764}$

b. $p(b|b) = \underline{0.01}$

$p(AB|AB) = \underline{0.0016}$

$p(O|O) = \underline{0.1936}$

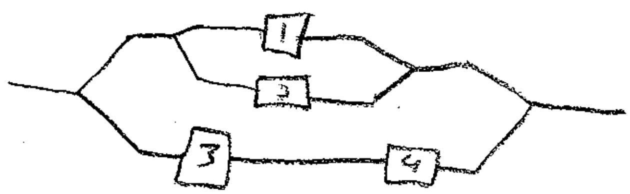
	A	B	AB	O
A	0.1764			
B		0.01		
AB			0.0016	
O				0.1936

c. $p(\text{matching 2 in a row}) = \sum P(\text{matches of each}) = 0.1764 + 0.01 + 0.0016 + 0.1936$

$p(\text{matching 2 in a row}) = \underline{0.3816}$

d. $p(\text{not matching 2 in a row}) = 1 - \text{ANSWR } c = 1 - 0.3816 = \underline{0.6184}$

5.20



ALL INDEPENDENT &

$P(\text{a given component works}) = 0.9$

$P(\text{a component fails}) = 0.1$

$P(\text{whole system works}) =$

$\Rightarrow \boxed{A}\ P(1 \text{ or } 2) = P(1) + P(2) - P(1)P(2) = 0.9 + 0.9 - 0.81 = 0.99 \Rightarrow A$

$\boxed{B}\ P(3 \text{ AND } 4) = P(3)P(4) = 0.81 \Rightarrow B$

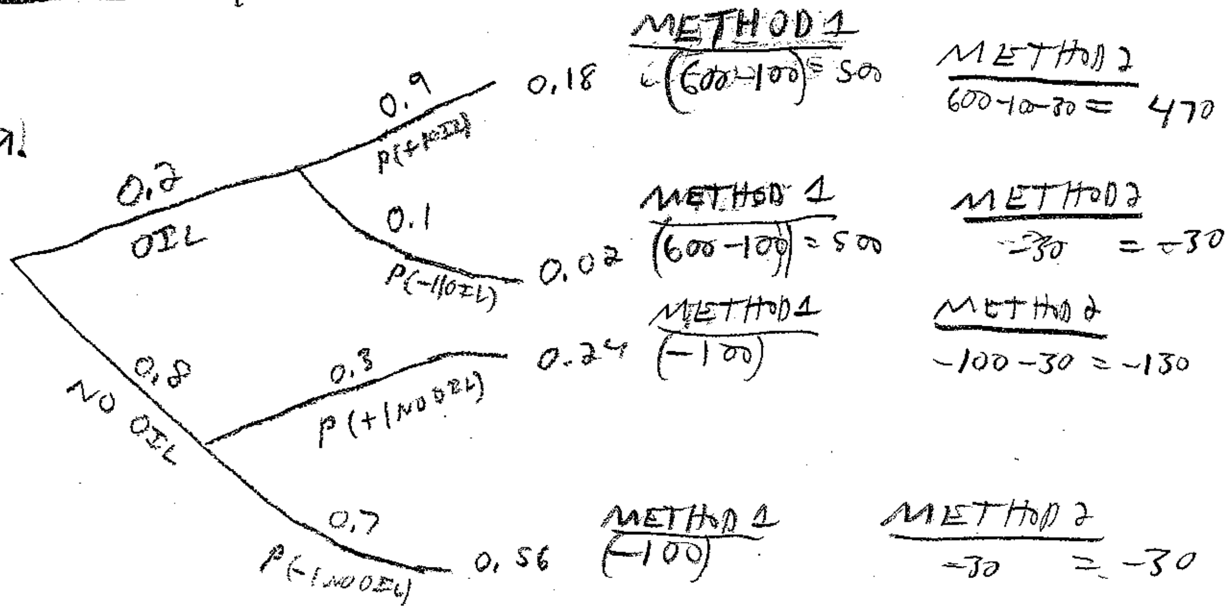
$P(A \text{ or } B) = P(A) + P(B) - P(A)P(B) = 0.99 + 0.81 - 0.8019 =$

$\boxed{0.9981}$

STT 351-002 HW #2 [redacted]

5.21 + $P(OIL) = 0.2$ $P(+|OIL) = 0.9$ $P(+|NO\ OIL) = 0.3$

a.

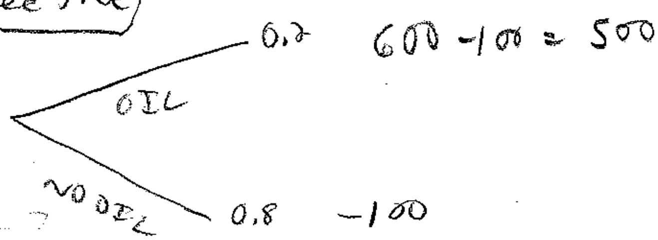


b. $P(+) = P(+|oil) + P(+|NO\ OIL) = 0.18 + 0.24 = 0.42$

c. $P(\overset{A}{OIL}|\overset{B}{+}) = \dfrac{P(+|OIL)\,P(OIL)}{P(+)} = \dfrac{(0.9)(0.2)}{(0.42)} = \boxed{0.4286}$

d. $\boxed{\text{see tree}}$

e.



f. $\boxed{\text{see tree}}$

g. JUST DRILL

$(0.2)(500) = 100$
$(0.8)(-100) = -80$

MEAN $= (100) + (-80) = \boxed{20}$

S.D. $\boxed{240}$

DRILL IF (+) TEST

$(0.18)(470) = 84.6$ $= 84.6 - 0.6 - 31.2 - 16.8$
$(0.02)(-30) = -0.6$ $= \boxed{36}$ $\boxed{207.47}$
$(0.24)(-130) = -31.2$
$(0.56)(-30) = -16.8$

I would pick "test first drill if (+)" because higher mean & lower distribution means less variation & more AVG. return.

Fin economics Markowitz Portfolio

STT 351-002 HW #2 [redacted]

5.26

$x=0$	1	2	3	4
0.08	0.15	0.45	0.27	0.05

a. Mean # of defects $= (0)(0.8)+(1)(0.15)+(2)(0.45)+3(0.27)+4(0.05) = \boxed{2.06}$

b. Variance: $\sigma^2 = (0-2.06)^2(0.08)+(1-2.06)^2(0.15)+(2-2.06)^2(0.45)+(3-2.06)^2(0.27)+(4-2.06)^2(0.05)$

Standard Deviation: $\boxed{\sigma = 0.9677}$ $\boxed{\text{Variance, } \sigma^2 = 0.9364}$

5.28

$p(x) = c(5-x) \qquad x = 0, 1, 2, 3, 4$

Find c & $p(x>0)$ $\qquad \int_0^4 c(5-x)\,dx = 1 = \left[5cx - \tfrac{1}{2}cx^2\right]_0^4 = 1$

$5c(4) - \tfrac{1}{2}c(4)^2 - \cancel{5c(0)} + \cancel{\tfrac{1}{2}c(0)^2} = 1 \qquad 20c - 8c = 1 \qquad 12c = 1$

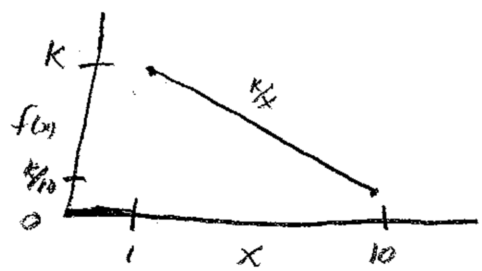
$p(x) = \frac{1}{12}(5-x)$ $\qquad \boxed{c = \frac{1}{12}}$

$p(x>0) = p(x=1)+p(x=2)+p(x=3)+p(x=4) = \frac{1}{12}(4)+\frac{1}{12}(3)+\frac{1}{12}(2)+\frac{1}{12}(1)$
$= \frac{10}{12} = \boxed{\frac{5}{6}}$

5.30

$f(x) = \begin{cases} k/x & \text{for } 1 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$

a.



b. $\int_1^{10} \frac{k}{x}\,dx = 1 \Rightarrow \left[k \ln x\right]_1^{10} = 1 \Rightarrow k\ln 10 - k\ln 1 = 1 \Rightarrow 2.3k = 1$

$\boxed{k = 0.434}$

c. $f(x>3) = \int_3^{10} \frac{0.434}{x}\,dx = (0.434)\ln 10 - (0.434)\ln 3 = \boxed{0.523}$

d) $f(2.75 < x < 3.25) = \int_{2.75}^{3.25} \frac{0.434}{x}\,dx = 0.434 \ln 3.25 - 0.434 \ln 2.75 = \boxed{0.0725}$

STT 351-002 HW #2 [redacted]

5.32 poisson mass function $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ where $x = 0,1,2,3,\ldots$ $\lambda \geq 0$

x = number of bacteria

λ = mean # of bacteria cells in the test tubes

a. $p(0) = \frac{e^{-\lambda}\lambda^0}{0!} = \boxed{e^{-\lambda}}$

b. $p(x \geq 1) = 1 - p(x=0) = 1 - e^{-\lambda}$

c. $p(x \geq 1) = 1 - e^{-\lambda} = 0.4$ $\quad -0.6 = -e^{-\lambda}$ $\quad 0.6 = e^{-\lambda}$

$-\lambda = \ln 0.6$ $\quad -\lambda = -0.51083$

$\boxed{\lambda = 0.51}$

$= \frac{1}{\lambda} = \boxed{1.958}$

5.38 $P(x \geq \cancel{t}+s \mid x \geq \cancel{t}) = P(t \geq s)$

Exponential Distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{with } \lambda \geq 0$$

$$P(X > t+s \mid X > t) = \frac{P(X > t+s \text{ and } X > t)}{P(X > t)} = \frac{P(X > t+s) \cdot p(x > t)}{p(x > t)}$$

$\frac{P(X > t+s)}{P(X > t)}$ $= \frac{e^{-\lambda s + \cancel{t}}}{\cancel{e^{-\lambda t}}} = \boxed{e^{-\lambda s}}$ $\quad P(x > s) = e^{-\lambda s}$

STT 351-002 HW #2 [redacted]

5.40

x:	10	15	20	25	30
$p(x)$:	0.10	0.20	0.30	0.30	0.10

y:	5	10	15	20
$p(y)$:	0.20	0.50	0.20	0.10

a. $\text{AVG } X = 10(0.1) + 15(0.2) + 30(0.3) + 25(0.3) + 30(0.1) =$

$1 + 3 + 9 + 7.5 + 3 = \underline{20.5}$

$\text{AVG } Y = 5(0.2) + 10(0.5) + 15(0.2) + 20(0.1) =$

$1 + 5 + 3 + 2 = \underline{11}$

b. $15 \Rightarrow (0.1)(0.2) = 0.02$

$20 \Rightarrow (0.1)(0.5) + (0.2)(0.2) = 0.05 + 0.04 = 0.09$

$25 \Rightarrow (0.1)(0.2) + (0.2)(0.5) + (0.3)(0.2) = 0.02 + 0.1 + 0.06 = 0.18$

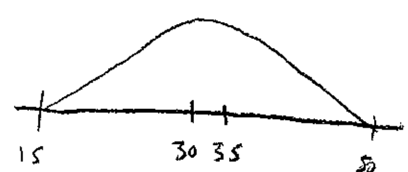
$30 \Rightarrow (0.1)(0.1) + (0.2)(0.2) + (0.3)(0.5) + (0.2)(0.3) = 0.01 + 0.04 + 0.15 + 0.06 = 0.26$

$35 \Rightarrow (0.1)(0.2) + (0.3)(0.5) + (0.3)(0.2) + (0.2)(0.1) = 0.25$

$40 \Rightarrow (0.3)(0.1) + (0.3)(0.2) + (0.1)(0.5) = 0.14$

$45 \Rightarrow (0.3)(0.1) + (0.1)(0.2) = 0.05$

$50 \Rightarrow (0.1)(0.1) = 0.01$



c. $p(x+y < 35) = 1 - p(x+y > 35) = 1 - \underbrace{(0.25 + 0.14 + 0.05 + 0.01)}_{\text{from above}} = \underline{0.55}$

d. Avg X + AVG Y from part a.

$23.5 + 11 = \underline{31.5 \text{ hours}}$

STT 351-002 | HW #2 | [redacted]

5.41 + $\{2,4,4,6\}$

a. 2,4 2,4 2,6

4,2 4,4 4,6

4,2 4,4 4,6

6,2 6,4 6,4

3 3 4

3 4 5

3 4 5

4 5 5

$p(3) = 4/12 = 0.33$

$p(4) = 4/12 = 0.33$

$p(5) = 4/12 = 0.33$

$SD \sqrt{\frac{(2-4)^2 + (4-4)^2 + (4-4)^2 + (6-4)^2}{4}} = \boxed{\sqrt{2}}$

b. $(3 * 0.33) + (4 * 0.33) + (5 * 0.33) = 4$

$SD = (X - mean)^2 p(x)$

$(3-4)^2(0.33) + (4-4)^2(0.33) + (5-4)^2(0.33) = 0.666 \quad \sqrt{0.666} = 0.816$ $\boxed{0.816}$

c. The formula would not work from page 229 because there is no replacement so it does not work. Sample size does matter in our case but not on pg 229.

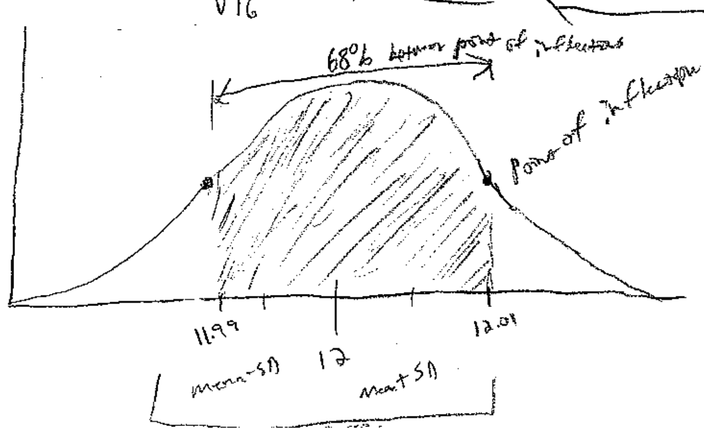
5.46 mean = 12 cm
SD = 0.04 cm

a. 12 cm 0.04 cm

b. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = 0.005$ cm

c. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.01}{\sqrt{16}} = 0.01$ cm 16 sample size

d.



5.52

mean of $50 = \mu_x$

SD $1.5 = \sigma_x$

a. $P(\bar{X}_9 > 52) \qquad \bar{X}_9 = \frac{X_1 + \dots X_9}{9}$

$$P\left(\frac{\bar{X}_9 - E\bar{X}_9}{SD\,\bar{X}_9} > \frac{52 - E\bar{X}_9}{SD\,\bar{X}_9}\right)$$

$$E\bar{X}_9 = \mu_{\bar{X}_9} = E\,\frac{X_1 + \dots X_9}{9} = \frac{50 + 50 + 50}{9} = 50$$

so $E\bar{X}_9 = 50$

SD of $\bar{X}_9 \qquad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{1.5}{\sqrt{9}} = \frac{1.5}{3} = \frac{1}{2}$

so $P(\bar{X}_9 > 52) = P\left(\frac{(\bar{X}_9 - E\bar{X}_9)}{SD\,\bar{X}_9} > \frac{52 - EX_9}{SD\,\bar{X}_9}\right)$

$$P\left(z > \frac{52-50}{\frac{1}{2}}\right) = P\left(z > \frac{2}{\frac{1}{2}}\right) = P(z > 4) = 1 - 1.0000$$

Table $P(z<4)$

$= \underline{0}$ $\underline{\text{Never}}$

b. $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{1.5}{40} = 0.0375$

$E\bar{X}_{40} = 50$

$\bar{X}_{40} = 52$

$P\left(z > \left(\frac{52-50}{0.0375}\right)\right) = P(z > 53.333)$ Convert to Table $\Rightarrow$ $\underline{\text{NO}}$ CHANCE