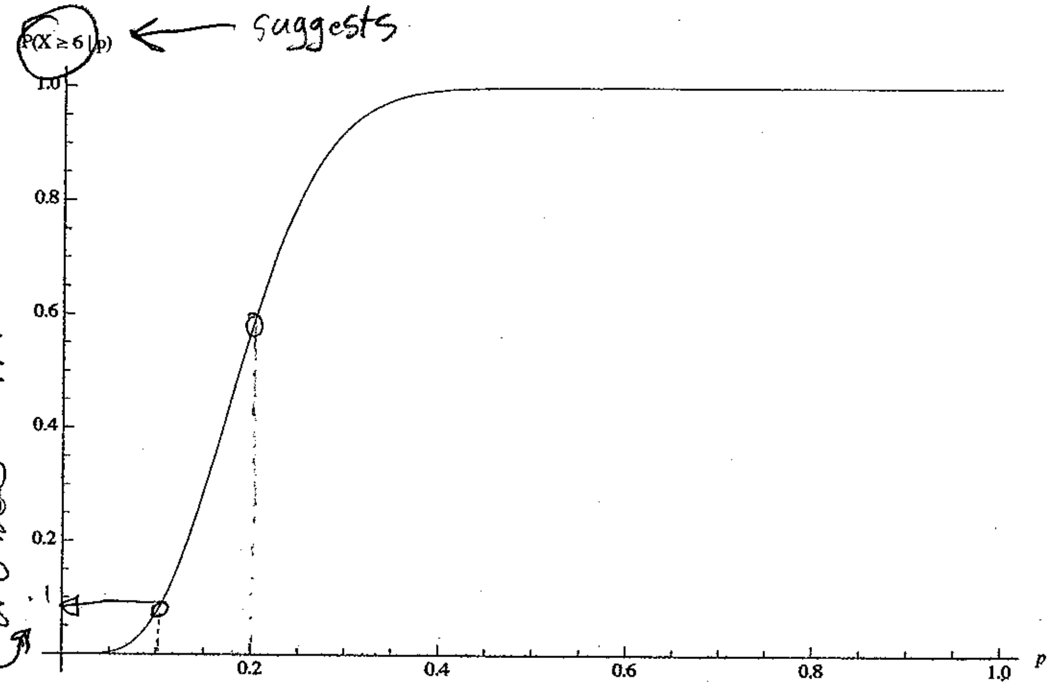
EXAM 3 KEY

Testing hypotheses.

1. Binomial p, H_0: {p = 0.1}. A sample of n = 30 items is selected from production. Let X denote the number defective in the sample.



a. Determine H_a for the types of tests we have been discussing. What property makes your choice the correct one? $H_a: p > .1$

This is an upper-tail test; that is, if the real rate of defectives is >.1 we wish to reject, if it is less that is not a problem.

b. Determine the probability of type I error α from the plot.

Looks to be ≈ .07

BONUS 1

c. If we were to increase n to 120, for the same α, how much steeper would the plot be (compare plots at their respective steepest points)?

Steeper by $\frac{\sqrt{120}}{\sqrt{30}} \simeq 2$

d. Which action, reject H_0, or fail to reject H_0, is taken by the above test if the number of defectives in a sample of 30 is 6.

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~~No rejection region or criteria for such is established.~~

~~It would seem reasonable, given that P(~~

Based on the note $P(X \geq 6 | p)$ I think the test has rejection region $X \geq 6$, so if $X = 6$ then reject H_0.

e. Determine the probability of type II error $\beta(0.3)$ from the table.

$X=5,\ p=.3 \rightarrow .0765948$

$\beta(.3) = .0765948$

Note that while $\beta(p)$ is the probability that you do NOT reject, and should have a $1 - p(\text{reject})$, the structure of the cumulative binomial has this accounted for...

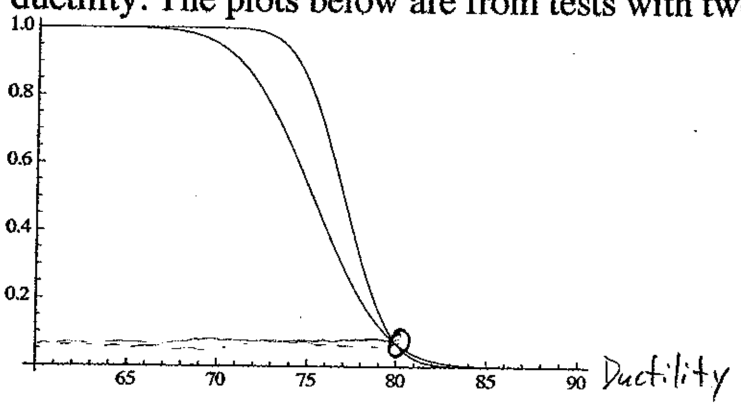
we ask the question, "what probability that we fail to reject, that is $X \le 5$, when $p=.3$" and the table tells us exactly that, the prob. that $X \le 5,\ p=.3$

n=30 Cumulative Binomial

x	p = 0.1	p = 0.2	p = 0.3
0	0.0423912	0.00123794	0.0000225393
1	0.183695	0.0105225	0.000312331
2	0.411351	0.044179	0.00211318
3	0.647439	0.122711	0.00931657
4	0.824505	0.255233	0.0301549
5	0.92681	0.427512	0.0765948
6	0.974173	0.60697	0.159523
7	0.992216	0.760791	0.281377
8	0.99798	0.871349	0.431518
9	0.999546	0.938913	0.588809
10	0.999911	0.974384	0.73037
11	0.999985	0.990507	0.840678
12	0.999998	0.996889	0.91553
13	1.	0.999098	0.959947
14	1.	0.999769	0.983063
15	1.	0.999948	0.99363
16	1.	0.99999	0.997875
17	1.	0.999998	0.999374
18	1.	1.	0.999838
19	1.	1.	0.999963
20	1.	1.	0.999993
21	1.	1.	0.999999
22	1.	1.	1.

t-test with σ known

2. A random sample of items is selected from production. Each item is scored x = ductility. The plots below are from tests with two different n and the same α.



a. By inspection of the plots determine

$\alpha = \sim .05$ $H_0 = 80$ $H_a = < 80$

b. (3 pts) Complete the form of the test statistic for n = 100 and $\sigma = 20$.

reject H_0 if $\dfrac{\overline{x} - (\ X_0\ )}{20/(\sqrt{100})} < -1.5$ ← from the graph this is 80

(3) ✓

LR (linear regression)

3. Data

	x	y	x^2	y^2	xy
	0	0	0	0	0
	3	0	9	0	0
	3	6	9	36	18
tot	6	6	18	36	18
avg	2	2	6	12	6

a. Calculate the sample sd s[x]. Do not reduce.

$\overline{x} = \frac{6}{3} = 2$

$\sqrt{\frac{(0-2)^2+(3-2)^2+(3-2)^2}{3-1}} = \sqrt{\frac{4+1+1}{2}} = \sqrt{3}$

b. Calculate the n-divisor (i.e. "plug-in" or "method of moments") estimator $\hat{\sigma}_x$ of σ_x. Do not reduce your answer.

$\sqrt{\frac{3-1}{3}} \cdot \sqrt{3} = \frac{2}{3}\sqrt{3}$

c. Calculate the correlation r[x,y]. **Reduce your answer.**

$\frac{\overline{xy} - \overline{x}\,\overline{y}}{\hat{\sigma}_x \hat{\sigma}_y} = \frac{6-2\cdot 2}{\sqrt{2}\cdot 2\sqrt{2}} = \frac{2}{4} = \frac{1}{2}$

$\sigma_x = \sqrt{\overline{x^2}-\overline{x}^2} = \sqrt{6-4} = \sqrt{2}$

$\sigma_y = \sqrt{\overline{y^2}-\overline{y}^2} = \sqrt{12-4} = \sqrt{8} = 2\sqrt{2}$

d. What fraction of $\overline{y^2}-\overline{y}^2$ is explained by regression of y on x?

$r^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ is fraction of Var Y explained by reg. on X

e. What is the numerical value of r[2x-9, 4y+11]? No calculation required.

1/2

#7

f. What is the numerical value of r[ŷ, y]? No calculation required.

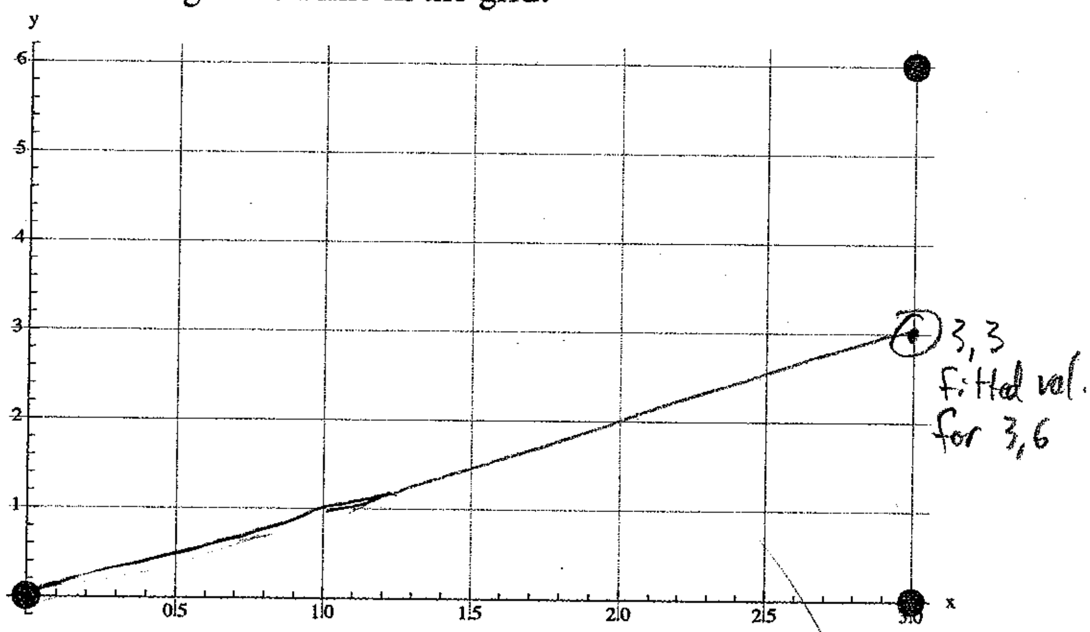
$|r| = |½| = ½$

$r[\hat{y}, y] = r_{mLR}$

g. Calculate the slope of regression $\hat{\beta}_1$ of y on x. **Reduce your answer.**

$\text{slope} = r \cdot \frac{\sigma_y}{\sigma_x} = \frac{2\sqrt{2}}{\sqrt{2}} \cdot ½ = 1$

h. Plot the regression line in the grid.



Passes through 0,0 and 3,3

i. Determine the fitted value ŷ for data point (3, 6) and also indicate it in the above plot. Don't neglect to show your method.

$\frac{\hat{y} - \bar{y}}{x - \bar{x}} = \text{slope} \qquad \frac{\hat{y} - 2}{3 - 2} = 1 \rightarrow \hat{y} = 1 + 2 = 3$

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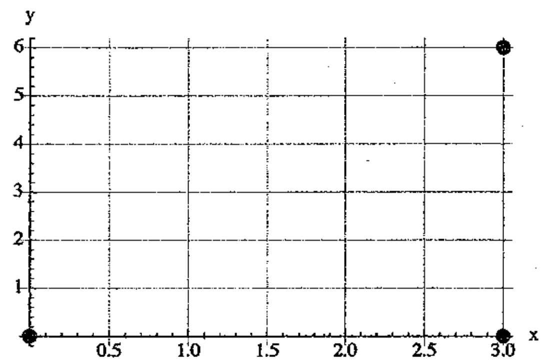
MLR

4. For (x, y) data



	x	y	x^2	y^2	xy
	0	0	0	0	0
	3	0	9	0	0
	3	6	9	36	18
tot	6	6	18	36	18
avg	2	2	6	12	6

a. Determine the *design matrix* (call it xx), and vector y, of a matrix re-formulation of LR of y on x.

$$\begin{pmatrix} 1 & 0 \\ 1 & 3 \\ 1 & 3 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 6 \end{pmatrix}$$

design xx   vector y

b. PseudoInverse[xx] = $\begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$. Determine $\hat{\beta}_0, \hat{\beta}_1$.

$$PI[xx].y = \begin{pmatrix} 1\cdot 0 + 0\cdot 0 + 0\cdot 6 \\ -\frac{1}{3}\cdot 0 + \frac{1}{6}\cdot 0 + \frac{1}{6}\cdot 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$

c. Determine fitted values $\hat{y}$ by feeding $\hat{\beta}_0, \hat{\beta}_1$ from (b) into xx. See that they agree with what you get by eye.

$0+1\cdot 0=0=\hat{y}_1$

$0+1\cdot 3=3=\hat{y}_2$

$0+1\cdot 3=3=\hat{y}_3$

and they are the same as plotted

d. Calculate $s[\text{resid}]^2$ from the residuals determined by eye and reduce.

$\frac{(0-0)^2+(0-3)^2+(6-3)^2}{3-1} = \frac{9+9}{2} = 9$

e. Inverse[Transpose[xx].xx] $\frac{n-1}{n-d} s[\text{resid}]^2 = \begin{pmatrix} 18 & -6 \\ -6 & 3 \end{pmatrix}$. Use this to determine 95% t-based CI for each of β_0, β_1.

$\beta_0: \hat{\beta}_0 \pm t_{1,.025}\sqrt{Var(\hat{\beta}_0)} = 0 \pm 12.706\cdot\sqrt{18}$

$\beta_1: \hat{\beta}_1 \pm t_{1,.025}\sqrt{Var(\hat{\beta}_1)} = 1 \pm 12.706\cdot\sqrt{3}$

Data and design

5.

		hardness	temp
y_i	1	x_{1i}	x_{2i}
6.3	1	13.8	2.9
3.7	1	10.4	1.6
4.6	1	15.7	2.3
7.7	1	17.5	5.2
5.2	1	12.8	3.6
4.1	1	14.7	2.8
6.3	1	15.9	5.1

Bonus .5

$\hat{\beta}$ = {3.61, -0.07, 0.79}. Estimate the mean response E Y for inputs temp = 3, hardness = 14. Do not reduce. Estimate must assume $E\,\varepsilon_i = 0$; under this assumption

$EY = 3.61 - .07\cdot 14 + .79\cdot 3$

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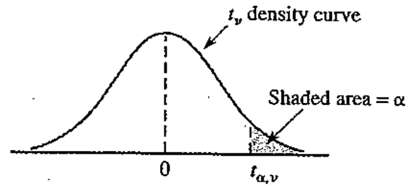
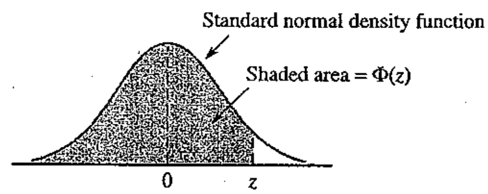
Table A.5 Critical Values for *t* Distributions



	α						
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998