Math

EXAM 3 PREP PARTIAL KEY $\quad y = 5x \quad$ (1)

5. SYSTEM $y = X\beta$

SOLVE SIMUL EQ^{NS}

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \underbrace{\begin{pmatrix} x_{ij} \end{pmatrix}}_{n \times n} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$$

$$\begin{pmatrix} x_{ij} \end{pmatrix}^{-1} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = I \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

STAT $\quad \underset{n\times 1}{y} = \underset{n\times d}{X} \underset{d\times 1}{\beta} + \underset{n\times 1}{\epsilon}$

Pseudoinverse (PRODUCES LS SOLN)

DENOTE $X^{(-1)}$

THEN $\underset{d\times 1}{\hat{\beta}} = X^{(-1)} y = \underbrace{X^{(-1)} X}\beta + \underbrace{X^{(-1)}\epsilon}_{\text{A LINEAR COMB OF COL'S OF } X}$

β

5b GIVEN

$$X^{(-1)} y = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{24} & -\frac{1}{24} & \frac{1}{12} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} = \hat{\beta} = X^{(-1)} y$$

$$1 = 0\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\right) + 4(0) = \hat{\beta}_0 = 1 \qquad \text{Ck's w/ pic}$$

$$\tfrac{1}{4} = 0\left(-\tfrac{1}{24}\right) + 2\left(-\tfrac{1}{24}\right) + 4\left(\tfrac{1}{12}\right) = \hat{\beta}_1 = \tfrac{1}{4}$$

5C. $\hat{y} = \underset{\text{NOT } X^{(-1)}}{X} \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 12 \end{pmatrix} \begin{pmatrix} 1 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

RESID $y - \hat{y}$ $\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2)

5d. $\hat{\sigma}^2(x) = \dfrac{\sum (x-\bar{x})^2}{n-1} = \dfrac{(-1)^2 + 1^2 + 0^2}{3-2} = 1$

5e. $\begin{bmatrix} 1 & -\frac{1}{12} \\ -\frac{1}{12} & \frac{1}{48} \end{bmatrix}$ IS GIVEN AS $\underbrace{\left(X^{t}_{d \times n} X_{n \times d} \right)^{-1} \frac{n-1}{n-d} \hat{\sigma}^2(\text{RESID})}_{\text{SAID TO BE}}$

ESTd COV $\hat{\beta}_i, \hat{\beta}_j$

EST OF Var $\hat{\beta}_0$ ESTd Var OF $\hat{\beta}_1$

CI

$\Rightarrow$ 95% CI (t-BASED) FOR β_0 (TRUE INTERCEPT)

$\hat{\beta}_0 \pm t^{\alpha = .025}_{DF\, n-d = 3-2 = 1}$ (ESTd SD OF $\hat{\beta}_0$)

$1 \pm 12.706 \sqrt{1}$

$P(\beta_0 \in t\text{-BASED 95\% CI}) = .95$

t-TABLE
$.025 = \alpha$
IS 12.706

FOR β_1: $\hat{\beta}_1 \pm t \sqrt{1/48}$

$1/4 \pm 12.706 \sqrt{1/48}$

USED NOTATION
$\widehat{\text{Var}}(\hat{\beta}_0)$

GEN'L $\hat{\theta}$
$\text{Var}\, \hat{\theta}$
$\widehat{\text{Var}}\, \hat{\theta}$

(3)

5h. $r_{MULT} = 1$ BECAUSE $r_{MLR} = r[\hat{y}, y]$

$\hat{y} \equiv y$ FOR PERF FIT

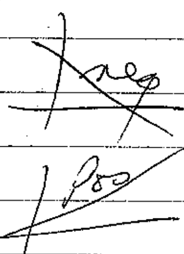
& $r[y, y] = 1$

5i. IMPORTANT ROLE FOR r_{MLR}

r^2_{MLR} = FRACTION OF $\overline{y^2} - \bar{y}^2 = .8^2$ ((64%))

ACCOUNTED FOR BY REGR ON $x_{n \times d}$.

$r_{MLR} = |r|$ r r



6.

6a GIVEN $\hat{\beta} = \{3.61, .79, -.07\}$
$\hat{\beta}_0 \quad \hat{\beta}_1 \quad \hat{\beta}_2$ EST

AT TEMP = 3, HARDNESS 14 $Ey = 3.61 + .79(3) + (-.07)14$ — ON AVG —

6b. TO INCL T^2 H^2 TH

1 2.3 1.57 2.3^2 1.57^2 2.3(1.57)

⋮

1 1.6 1.04 1.6^2 1.04^2 1.6(1.04)

Now $d = 6$

$n = 7$

$DF = 7 - 6 = 1$

6c. SUPPOSE (NOT!!) r_{MLR} ORIG DESIGN (1 Term Hardness)

IS $r_{MLR} = 0.78$ $r^2_{MLR} \approx .64$

AND FOR EXTENDED DESIGN $1\ T\ H\ T^2\ H^2\ TH$

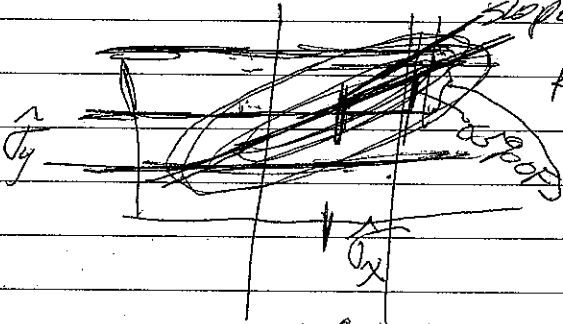
GET $r_{MLR} = 0.81$, $r^2_{MLR} \approx .64$

LITTLE DIFFERENCE IN r^2_{MLR} IN SPITE OF SO

MANY ADDIONAL VARIBLES.

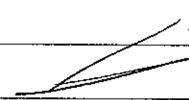
6d. SAME r_{MLR} (DOES NOT DEPEND UPON LOCATION + SCALE OF ANY OF THE VARIBLES (UNLESS YOU MULT BY "0")

7. a.



SLOPE $\hat{\sigma}_y/\hat{\sigma}_x$

REGR GOES THRU VERT STRIP AVGS



slope $\hat{\sigma}_y/\hat{\sigma}_x$

SLOPE REGR $r\frac{\hat{\sigma}_y}{\hat{\sigma}_x}$

68%?
"BY EYE"

APPEARS (MY CARBON) THAT PERHAPS $r \approx .65$ SINCE REGR LINE SLOPE IS AROUND 65% OF σ_y/σ_x SLOPE

(MAYBE PLOT IN 7a HAS $r \approx .55$??)

GIVE YOUR NUMERICAL EST^S $\hat{\sigma}_x$, $\hat{\sigma}_y$ (BY EYE)

MAYBE $\hat{\sigma}_x = 2$, $\hat{\sigma}_y = 10$??