Supplemental review of CI and testing.
12-01-08

We've studied a variety of confidence interval (CI) procedures. Their objective, when used at 95 % confidence, is to establish a margin of error for a statistical estimator: $\hat{\theta} \pm (\text{t or z}) \sqrt{\hat{\text{Var}}(\hat{\theta})}$. In this notation $\hat{\theta}$ denotes an estimator of some parameter θ and $\sqrt{\hat{\text{Var}}(\hat{\theta})}$ denotes the estimator of the standard deviation of estimator $\hat{\theta}$. When using bootstrap we typically never see (t or z) or $\sqrt{\hat{\text{Var}}(\hat{\theta})}$ since the bootstrap method delivers an estimator of the entire package (t or z) $\sqrt{\hat{\text{Var}}(\hat{\theta})}$.

CI

Here is a brief summary of most CI studied in this course (ex indicates that the CI is exact with perfect calculations, otherwise the CI is approximate as n -> ∞, and other assumptions including N - n -> ∞ when sampling finite populations).

$\hat{\theta} \pm (\text{t or z}) \sqrt{\hat{\text{Var}}(\hat{\theta})}$ **caveat**

$\overline{x} \pm t \frac{s}{\sqrt{n}}$ ex **eq-pr w/r sam of n from *normal* pop**

$\overline{x} \pm z \frac{s}{\sqrt{n}}$ **eq-pr w/r sample of n**

$\overline{x} \pm z \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$ **eq-pr without/r sample of n**

$\hat{p} \pm z \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$ **eq-pr w/r sample of n**

$\hat{p} \pm z \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$ **eq-pr without/r sample of n**

$\overline{x}$-$\overline{y}$ ± z $\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$ **indep x, y sam, eq-pr, w/r**

$\hat{p}_x$ - $\hat{p}_y$ ± z $\sqrt{\frac{\hat{p}_x (1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y (1-\hat{p}_y)}{n_y}}$ **same as just above**

$\hat{\theta}$ ± 95-th (or other) percentile of $|\hat{\theta}^* - \theta|$ **as approp**

$\overline{x}$ ± z $\frac{\sqrt{\sum_{i=1}^{K} W_i s_i^2}}{\sqrt{n}}$ **prop'l eq-pr w/r strat sam**

$\hat{p}$ ± z $\frac{\sqrt{\sum_{i=1}^{K} W_i \hat{p}_i (1-\hat{p}_i)}}{\sqrt{n}}$ **prop'l eq-pr w/r strat sam**

($\overline{y}$+ ($\overline{x}$-μ_x) r $\frac{\hat{\sigma}_y}{\hat{\sigma}_x}$) ± z $\frac{s_y}{\sqrt{n}} \sqrt{1 - r^2}$ **eq-pr w/r prs (x, y)**

$\hat{\beta}_i$ ± z $\sqrt{(i,\ i) \text{ entry of } (x^{\text{tr}}\, x)^{-1}\ \frac{n-1}{n-d}\ s[\text{resid}]^2}$ **with { ϵ_i} ind N(0, σ^2)**

$\hat{\beta}$ = PseudoInverse[x].y

$\hat{y}$ = x.$\hat{\beta}$

resid = y - $\hat{y}$

Tests from CI

Here is a brief description of how CI may be used to test hypotheses of the kind studied in tis course. The idea is a simple one. A CI is trying to locate (cover) a parameter θ. If it is a 95% CI then P(CI covers θ) ~ 0.95. So P(CI misses θ) ~ 1 - 0.95 = 0.05. If could devise a test of (for example) the null hypothesis that $\theta = 17$ versus the two-sided alternative hypothesis $\theta \neq 17$ which

rejects $H_0 : \theta = 17$ if CI fails to cover 17.

If truly $\theta = 17$ such a test commits type I error precisely when CI fails to cover 17. This has probability 0.05 as above. So $\alpha = 0.05$ for such a use of CI to test.

If, instead, we wish to perform a one-sided test

H_0: $\theta = 17$ versus H_a: $\theta > 17$

we could harness the CI in the following way:

reject H_0 if CI falls entirely to the right of 17.

For this test, α = (1-0.95)/2 = 0.025 since the 0.05 probability of having the CI fail to cover 17 is about equally divided between missing to the left or missing to the right.

Testing

H_0: $\theta = 17$ versus H_a: $\theta < 17$

we would

reject H_0 if CI falls entirely to the left of 17.

Once again, $\alpha = 0.025$.

0 - 1 data and tests

We might make a test for

H_0: p = 0.1 versus H_a: p ≠ 0.1

by

rejecting H_0 if CI $\hat{p} \pm z \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$ fails to cover 0.1.

Then $\alpha \sim 0.05$. but that is not the preferred test. If p = 0.1 then the population sd is $\sqrt{0.1 \times 0.9}$ and need not be estimated by $\sqrt{\hat{p}\,(1 - \hat{p})}$. The preferred test

rejects H_0 if $\hat{p} \pm z \frac{\sqrt{0.1 \times 0.9}}{\sqrt{n}}$ fails to cover 0.1.

Two points:

a. The preferred test looks like it uses a CI, but does not.
b. The preferred test more closely achieves $\alpha \sim 0.05$.

The one-sided counterpart

H_0: p = 0.1 versus H_a: p > 0.1

reject H_0 if $\hat{p} \pm z \frac{\sqrt{0.1 \times 0.9}}{\sqrt{n}}$ falls entirely right of 0.1

is preferred over the CI test as well. It more accurately achieves $\alpha \sim 0.025$ than does the one-sided CI test.

Comments.

z, t may take values other than 1.96, or the t-values related to 95% confidence. As pertains to using 99% CI to z-test we have

α ~ 1 - 0.99 = 0.01 in two-sided test

α ~ (1 - 0.99) / 2 = 0.005 in one-sided test.

As for β, we limit ourselves to the types of plots and table uses found on exam 3.

Any of the CI can be used to test in the way just outlined.