# MA 516 / STAT 541. Advanced Probability and Options, with Numerical Methods.

Fall 2005. Frederi Viens, Associate Professor of Statistics and Mathematics.
Classroom and times: Tu Th 3:00-4:15pm in REC 309

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Office hours: Tu 1:30 - 3:00 and Th 9:30-10:30, in MATH 504.

**Textbooks.**

**Bjork.** *Arbitrage theory in continuous time.* Author: Tomas Bjork. Oxford U.P., 1998/2004.

**LL.** *Introduction to Stochastic Calculus Applied to Finance.* Authors: D. Lamberton and B. Lapeyre. Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.

**Suggested additional reading.**

**WHD.** *The mathematics of financial derivative. A student introduction.* Authors: P. Wilmott, S. Howison, J. Dewynne. Cambridge U.P. 1995. Chapters 7 to 10.

**CS.** *Implementing derivative models.* Authors: L. Clewlow, Ch. Strickland. John Wiley and Sons, Ltd., 1998.

**C** *Statistical Analysis of Financial Data in S-Plus.* Author: René A. Carmona. Springer Texts in Statistics, January 2004. ISBN: 0387202862.

**Prerequisite:** MA 515 / STAT 540, Math of Finance.

**Required future enrollment:** STAT 598 I. Computational Finance Seminar. **ATTENDANCE IS MANDATORY. SEMINAR ONLY RUNS IN THE SPRING SEMESTER.**

**Suggested concurrent and future enrollment.**

- MGMT 614. Portfolio Management.
- MGMT 641. Options and Futures.
- MGMT 642. Security Analysis.
- ECON 673. Times series econometrics.
- ECON 676. Economics of Uncertainty.
- IE 581. Simulations Design and Analysis.
- MGMT 643. Financial Risk Management.
- MA 596 U / STAT 598 W. Design and Analysis of Financial Algorithms.
- MGMT 690S. Pricing Strategy and Analysis.
- IE 590 A. Financial Engineering.
- IE 690 D. Advanced Financial Engineering.

**Course description**. This is the second course in a two-course sequence on the mathematics of finance, and especially on option pricing. The material will be divided in two parts. First, we will cover theoretical issues regarding: (i) interest rate term structure models; (ii) American options and stochastic optimal stopping; (iii) finite difference methods. Then we will examine in detail the numerical methods used to solve the partial differential equations and inequalities that determine the prices of options, including the Binomial, Trinomial, Monte-Carlo, and finite difference methods.

**Grading scheme.**

- **Quizzes (10 to 15 %).** During the first (theoretical) part of the course (first two thirds), two or three in-class 'quizzes', lasting about 60 minutes each, will test your grasp of the material covered in class since the previous quiz, although some quizzes may be more comprehensive. See the note below about plagiarism[1], which will not be tolerated.
- **Theoretical homework (5 to 10 %).** We will hopefully have a grader for the this course, which means that selected homework problems in the textbooks used (Bjork, LL), and possibly other assignments, will need to be turned in, and will be graded. There will be theoretical homework problems assigned mainly throughout the first two thirds of the course. While it is acceptable to work in groups on homework problems, each student must turn in a separate assignment, and identical solutions are NOT acceptable[1]. Your homework must reflect YOUR understanding of the material.
- **"Final" exam (30%).** The "final" exam will be a 80-minute in-class exam administered towards the 9th or 10th week of classes on the theoretical material covered in the lectures up to that point.
- **Simulations homework (15%).** During the last third of the course, you will be required to code up many of the algorithms that we will encounter for option pricing. You will be encouraged to write your code in C++, but any language and/or program is acceptable. Note however that since C++ is the language of choice in the industry, it is to your advantage to learn it and use it. Some of these coding assignments may be accompanied by theoretical questions. The books CS, LL, and WHD contains many algorithms which you may use.
- **Class project (15%).** You will be required to turn in one class project/paper. Project topics will combine questions that we cannot cover in class for lack of time, with the practice of their associated numerical methods. You will be required to illustrate the numerical methods by coding some computer simulations, and by presenting the results of your simulations in a critical way. The written projects will be due near the end of the semester. We will identify small groups of students to work together on a single project. Each group will submit a single written report. There will be no group in-class presentation. IT IS TO YOUR ADVANTAGE TO START THINKING ABOUT A TOPIC TO WORK ON AS EARLY AS THE 9TH WEEK OF CLASSES.
- **Oral examination (15%).** You will also have to take an individual oral examination in front of one or more faculty members in the Computational Finance program, including Prof. Viens. The faculty member(s) will ask questions which the student will be required to answer orally and/or on the blackboard, as appropriate. Most questions will test the student's understanding of the previous course 515/540, and of the first (theoretical) part of the current course 516/541. Students not seeking a CF MS degree will only be asked such questions. Occasionally, advanced Ph.D. students seeking a CF MS degree may have to give a special presentation and may be asked other types of questions. **Note that in the Department of Statistics, receiving a passing grade on this oral exam is a necessary condition for graduation with a CF MS degree**. Students who are seeking the CF specialization may be asked to retake 516/540 and/or the oral examination if their oral performance is particularly unsatisfactory.

[1] Plagiarism is the act of presenting someone else's work as your own. This includes finding the answer to a given problem in a book, in someone else's assignment, or requesting the answer from someone, and copying from it. Contrary to popular belief, a correct solution to a given mathematical problem is almost never unique, and plagiarism in a mathematical assignment is very easy to detect.

**Outline of the course**

- The term structure of interest rates: bond and option pricing (Bjork Ch 20 - 25)
  - coupon bonds
  - short rate models, with pricing equations
  - examples of martingale models: Vasiček, Ho-Lee, CIR, Hull-White, ...
  - forward rate models in the Heath-Jarrow-Morton framework
  - change of numeraire, with martingale measure
  - LIBOR and swap market models
- American options (LL Ch 2 and WHD Ch 7)
  - stochastic optimal stopping
  - pricing and hedging the American put option
  - continuous-time setting, and free boundary problems for American options
- Introduction to the theory of finite difference methods (LL Ch 5, WHD Ch 8,9)
  - resolution of parabolic equations, European options
  - resolution of parabolic inequalities, free boundaries, American options
- Numerical methods for solving the pricing partial differential equations.
  - Numerics for the finite difference methods (LL Ch 5)
  - Binomial methods (CS Ch 2)
  - Trinomial trees: from binomial trees to finite difference methods (CS Ch 3)
  - Monte-Carlo methods (CS Ch 4)
  - Methods for exotic and path-dependent options (CS Ch 5)
  - Special methods for interest-rate models (CS Ch 6-10)